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THE ARITHMETIC TEACHER

Volume 8, number 6 OCTOBER **1961**

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Numbers, sets, and counting — *Truman Botts*

Sets and numbers in grade one, 1959-1960 — *Patrick Suppes and Blair A. McKnight*

Some new approaches to old arithmetic — *Emilio Rivera*

Mental arithmetic — *Robert H. Koenker*

Multiplication facts and the role of 1 in the elementary school, the status of research, the aftermath of the annual business meeting—financial report, bylaws, committees, and committee reports

AN OFFICIAL JOURNAL OF *The National Council of Teachers of Mathematics*

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THE ARITHMETIC TEACHER
an official journal of
The National Council
of Teachers of Mathematics

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THE NATIONAL COUNCIL
OF TEACHERS OF MATHEMATICS
1201 SIXTEENTH STREET, N.W.
WASHINGTON 6, D.C.

Second-class postage paid at Washington, D.C., and at additional mailing offices.

THE ARITHMETIC TEACHER is published monthly eight times a year, October through May. The individual subscription price of $5.00 ($1.40 to students) includes membership in the Council. (For an additional $3.00, or $1.00 to students, the subscriber may also receive THE MATHEMATICS TEACHER.)

Institutional subscription: $7.00 per year for each journal. Single copies: 85¢. Make remittance payable to The National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington 6, D.C. Add 25¢ per year for mailing to Canada, 50¢ per year for mailing to other foreign countries.

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As we read

E. W. HAMILTON *Associate Editor*

This issue greets many new readers as well as a host of old ones. Students, undergraduate and graduate, as well as many new recruits to the teaching profession and veterans newly pledged to more emphasis on arithmetic, will be scanning our pages with hope and interest.

A capacity crowd heard Professor Sueltz deliver the address, "A Time for Decision," in Chicago last spring. For the record and for the benefit of others who did not hear him, it is reported in full in this issue. The address is both a summary of the *status quo* and a compilation of questions still demanding answers. Both are worth careful consideration as they are proposed in dead earnest by a man of wisdom and long experience and by the man whose care and effort built this magazine.

Particularly worth noting from the student's point of view is the implication that some questions that were answered by research a generation ago seem to be still unanswered in the present day, thus raising in a specific setting the question of relevance of research to background philosophy. Dr. Sueltz's comments on the difference between children and adults with respect to interests, goals, and methods of learning, particularly with regard to the time, place, and manner of generalizing, deserve sober reflection. Those mathematicians, may their shadows never grow shorter, who are willing to help mathematize arithmetic for the elementary schools need especially to be careful on these points. Also, many teachers and students who take advanced work in institutes and seminars are in danger of adopting from these mature courses a too mature deductive approach without sufficient modification by inductive exploration and the gradual accumulation of concepts and definitions. Ill-directed enthusiasm, lacking sufficient wisdom, could cause us to miss some very desirable goals that now seem to be in sight. It is this wisdom and leadership that Dr. Sueltz is demanding lest we "lose the ball" so to speak.

As an example of some of the new experimental materials referred to in the above speech consider "Sets and Number in Grade One" by Suppes and McKnight. Note that this is not being proposed as just something new to do but has a rationale behind it. It is being subjected to several kinds of objective evaluation in addition to the impressions and judgments of the participating teachers and school officials.

The language, notation, and concepts being built by easy stages and in natural settings with these young children are those employed in the article by Botts which deals with the real number system. This later article, as you will see, deals with some of the subtlest problems of logic that man has ever devised for himself, yet the language and notation are simple.

Just how far we can go toward improving ourselves by mere determination to

(*Continued on page 286*)

A time for decision*

What shall be the mathematics in the elementary school?

BEN A. SUELTZ

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Professor Sueltz is chairman of the department of mathematics at State University College of Education. Also, he is the first and immediate past editor of THE ARITHMETIC TEACHER.

Mathematics is queen of the sciences and arithmetic is queen of mathematics. She often condescends to render service to astronomy and other natural sciences, but under all circumstances the first place is her due." So wrote the great mathematician Gauss more than one hundred years ago. The queenly state is a high and noble estate with qualities such as those of beauty and charm, of authority and power, and of service to her people. This is the queen of the sciences who begins to serve in the elementary school.

The duality of mathematics

The elementary student, the layman, and perhaps many teachers confuse the machinery of mathematics with the essence of mathematics. That is, their view is apt to be limited to such things as $2+2=4$, $A=r^2$, and sin $A=y/r$ as these are used in the solution of problems. This is the machinery of mathematics. The essence of mathematics deals with the study of structures and relationships, such as the number system, and is characterized by a spirit of inquiry. This duality between the mechanics and the spirit of mathematics is historic. It is noted more than two thousand years ago in *logistic*, or reckoning, and *arithmetic*, or the science of numbers. In my own state of New York, more than three hundred years ago, the royal governors for New Amsterdam prescribed that schoolmasters should teach both cyphering and arithmetic. In more recent times, we have used the term "arithmetic" to identify the mathematics of the elementary school. But the duality of computation versus the science of numbers has continued, and this was wholesome because each aspect of the subject has led to developments which are useful.

In the present century, we have had the usual excesses. These will be recalled by such terms as "tool subject" and "social utility" which tended to emphasize abstract computations with a minimum of understanding of the mathematical relationships which are the very essence of making arithmetic an educational subject rather than a routine which might better be done by a machine. The educational psychologists had a dominant role in not only the methods of learning but also in the selection of subject matter in the period of the 1920's, '30's and early '40's.

Recall that noble research that established the relative difficulty of the number combinations and how textbooks were written in terms of this invalidly applied research. Recall also the effects of the studies which showed what arithmetic

* Presented at the Thirty-ninth Annual Meeting of the National Council of Teachers of Mathematics at Chicago, Illinois, on April 6, 1961.

people used and those that "proved" that a certain mental age was necessary for learning such a process as long division. And the role of the progressive must be mentioned because it was such a person who proclaimed that "of course we will teach division in the first grade if we encounter an experience in which division is used." We must view the past in the hope that similar mistakes are not made in the immediate future because this is now a time for decision.

The terminology "mathematics for the elementary school" came into use in many states as a protest against the very narrow view of arithmetic held by educational psychologists, curriculum workers, and teacher-trainers in the professional schools of education. The excesses and errors of judgment were made by people who did not consider all the factors involved, the most important of which are: (a) the subject matter of mathematics, (b) the child who is to learn the subject, (c) the teacher, and (d) the needs of society or the needs of the individual in society. It is in terms of these factors that we must make intelligent decisions.

The citizen and his society

Let us address ourselves to these topics in reverse order, beginning with the role of a citizen in his society both current and future. It is a dynamic, changing society. This factor cannot be viewed as a constant, either in terms of the individual or of society because we have many societies and many roles of individuals therein. The most common demand of society for its members is that they shall be able to think. Hence, they must have the equipment for thinking and the habit of thinking.

The equipment for thinking mathematically involves several stages: (a) the understanding of things mathematical, (b) the recognition of these in diverse situations, and (c) the desire, or will, to think and to draw and test conclusions.

The best way to assure thinking, currently and in the future, is to provide practice throughout the school experience. Here we must understand that numbers and computations are part of the machinery of thinking mathematically, but not the real essence thereof. We must also admit that reflective thinking is not automatic for the more gifted in our society and to be denied to those of lesser ability. For example, Mrs. G., who has an IQ of at least 120, had a recipe that called for $\frac{3}{4}$ cup of oil. But she wanted only half that amount, so she poured $\frac{3}{4}$ cupful into the measuring cup, then with her thumb marked her estimate of half that amount and poured back into the bottle until her thumbline was reached. And Mr. B., whose IQ is 83, when confronted with finding $\frac{1}{2}$ of $\frac{3}{8}$, reasoned to $1\frac{1}{2}$ eighths, and for his purpose that was sufficient. Perhaps half of our population who finished the study of arithmetic some thirty or forty years ago and learned by memorizing facts and procedures are today woefully weak in thinking mathematically and also very poor at the same computations at which they once excelled. Memory of facts is very fleeting.

Money is perhaps our most common commodity with which the intelligent member of society should be able to think. Also, money is an item that people seem to desire and to desire in amounts larger than most possess. It isn't really the money, but that which money will obtain that people crave. So, let us look briefly at money and thinking. Yes, it is valuable to be able to add and subtract and multiply and divide money, but that is largely record-keeping and not essentially obtaining more money or stretching its significance. Money and percentage are closely associated. It is appalling how many women will worship the 3% or 4% annual return on a savings account and yet pass up savings of 5% weekly in marketing. The time factor seldom suggests itself, and hence these purchasing agents for the American household are limiting their own

resources. Consider three little situations:

- A return of $1 per year on an investment of $21 represents a per cent.
- A price of 27¢ each or 2 for 49¢ is a certain per cent of difference.
- A reduction of 30% versus a reduction from $89 to $69.

Again it should be noted that thinking is based upon understanding basic concepts and principles and that computation is but a step in the intelligent thinking. Most of the items cited involve fractions, but in a way different from the usual textbook procedure.

The citizen in society encounters much mathematics which is not usually included under the term "arithmetic." The realms of scale, perspective, proportion, symmetry, harmony, and balance, are often identified with geometry, and these serve important roles in the cultural and aesthetic lives of the thinking citizen. Similarly, the understanding and appreciation of music and the structure of logical sequences are essentially mathematical. In baseball, we often say, "You can't hit it if you can't see it," and likewise the functioning citizen cannot sense mathematics, understand it, and think with it if he doesn't see it.

The child and mathematics

Let us look briefly at the child who must learn mathematics in the elementary school. It is a mistake to generalize *the* child because there are very many kinds of children and they are different in many ways and will have futures that are markedly different. If we could admit that a child is essentially a curious person who likes to explore, we would then have the keynote to learning mathematics. He is, in fact, similar to the adult mathematician and creative worker in that he naturally explores with facts and ideas and establishes conclusions. This is an inductive process. He is unlike the adult mathematician in that he is not concerned with a deductive proof of his conclusion and with one that covers all cases. Thus it would seem to be a mistake to ask the child to learn deductively; that is, learn a generalization and then seek its many applications.

Children generally are more concerned with the image or idea than with a precise statement thereof. The adult mathematician is greatly concerned with a parsimonious statement that says precisely what is intended. With children, we should be concerned with the nurture of ideas and the gradual refinement of their expression. In recent years, certain writers have become exercised over expressions, such as "5 pennies make a nickel," "3/4 is the same as 9/12," and "divide the numbers." Someone has suggested the introduction of a new symbol for "equivalent" to be used for equality in many cases. It should be remembered that children are growing in maturity and that we must recognize levels of sophistication of language and expression as well as the understanding of concepts and principles.

We must recognize that some things should be open to speculation and discovery, while others are probably best learned directly or authoritatively. That is, it would seem unwise to speculate about a symbol to use for the value eight (8) or for equality. But it might be very wise to speculate and discover the totality of combining 8 and 7 items because here the method of thinking, of discovery, is useful in many other situations and may be reused instead of relying upon pure memory. Furthermore, the spirit and method of discovery may be the more lasting and the more valuable part of the total learning process.

The role of the teacher

Because students differ in so many ways—in background of learning and experience, in interests, in attitudes and emotions, in will and determination, and in mental ability—it would seem unlikely

that they should all respond to the same modes of learning. Here we have a tremendous challenge, and the role of the teacher is paramount. It is the teacher who sets the stage or creates an atmosphere or climate for learning; it is he who selects and adjusts methods of learning for a class, for groups, and individuals; it is the teacher who selects the subject matter and the way in which it is learned; it is the teacher who sets goals and standards. Of course, outside agencies issue courses of study and advise and even dictate, but the teacher is the dominant factor.

While teaching in the elementary school is not like the expository lecture one finds in the graduate school, certain factors are common to both: (a) a teacher who thoroughly understands his subject and its many applications and ramifications can do a better job than one who is uncertain in scholarship, and this is revealed in many ways; (b) both should have a clear understanding of the goals or values the student should derive, but here the graduate lecturer frequently (especially the young one) fails in perspective; (c) at both levels it is wise to anticipate the hard or rough spots and to build the development accordingly; (d) both the elementary-school teacher and the graduate-school lecturer should be concerned with *understanding* and *significance* and the ability to think toward a conclusion instead of merely learning to repeat the development of another person.

There are essential differences in the work of an elementary-school teacher and that of a graduate-school lecturer. (I use the term "lecturer" instead of "teacher" for the graduate school because it often seems to apply at that level better than teacher, which has a much richer connotation.) The elementary-school teacher must have competencies in many fields, whereas the instructor in the graduate school is now expected to have a competence in perhaps geometry, algebra, or analysis. The students who come to the elementary school *must go there*, while those in the graduate school *elected to do so*. The range of abilities, interests, and backgrounds is much greater in the elementary school. The teacher at the elementary level has certain external pressures, such as courses of study and standardized tests, whereas the graduate lecturer is almost a free agent. Furthermore, in the elementary school the teacher directly supervises and aids the student in his study and practice, while in the graduate school the individual student must dig and the instructor is not available to assist and perhaps to elucidate better those items which he had assumed.

Regarding the subject matter of the elementary school

With mathematics generally recognized as the structural core of our industrial and technological society, and with the tremendous international competition current, mathematics at all levels enjoys a prestige undreamed of a mere decade ago. But this prestige and respect carry with them a great responsibility. It would be so easy to develop a sophisticated program for the elementary school that might be ideal for the one person in perhaps 10,000 who will become a mathematician or researcher in some area of science and not only fail to provide the things needed by the other 9,999, but also be harmful to them in their attitudes and emotions. The general public would soon turn away from mathematics, and society would be the loser.

Likewise, if a program were established that featured only computational arithmetic and written problems, not only would the one in 10,000 lose, but also the other 9,999. There is more than typical arithmetic available for the elementary school.

In recent years, certain trends are discernible. Programs are being planned for the kindergarten because there is ample evidence that children of this age not only can learn but also find pleasure in learning

things quantitative. In grades one and two, we find not only more arithmetic but some simple study of geometric form and a greater certainty of learning than just a few years ago. The spirit of "play school" and "happiness with ease" is being discarded in favor of satisfaction in achievement. For children of ages 5 through 8 we are finding that the once-hallowed doctrines of "readiness" and "attention span" may have been founded upon inadequate, if not spurious, educational research.

There seems to be a trend toward increasing the amount of time spent on mathematics in the elementary school. For the past 25 years, this has averaged about 12½% of the school day. A century ago, it was probably 25%, and in Europe it is much higher than in the United States. School people are often surprised and dubious when they hear that 7- and 8-year-olds in Europe are spending as much as 60 minutes per day on mathematics. In grades 3 and 4, schools seem to be tightening expectations and advancing the content beyond the levels of a decade ago. In grades 5 and 6, the trends noted for previous grades are sustained and in many schools new approaches and new materials are being introduced. These will be discussed later.

The single greatest trend in the whole elementary school is one of attitude toward mathematics. The older pattern of "explain-practice-perform" is being replaced with a new spirit or attitude, a spirit of adventure, of speculation, thinking, discovery leading to understanding, and self-projected learning. It is this same spirit of adventure that has established a favorable climate for experiment.

Most of you are familiar with the larger and well-publicized experimental programs that introduce sets, geometry, algebra, mathematical properties, such as closure and the commutative, and principles of number systems, in a way and at a level not usually found in the elementary school.

All of these experiments are sincere attempts to develop a deeper understanding of mathematics and to make it more significant to the boys and girls so that their lives are enriched thereby. The pattern seems to be one of starting with abstract values (numbers), making discoveries with and about these values, and then perhaps applying the discoveries to the social and physical world. New terminology and new symbolism is introduced as desired and required. Already it has been demonstrated that children are not afraid of new symbols and ideas. But, of course, that is not startling because all of education, of growing up and older, is a process of learning new things. I cannot refrain from one amusing story. A little girl reported to her mother, "Mother, we learned *curly brackets* today. It's more fun than arithmetic."

We do not have an adequate account of the many, perhaps thousands, of experimentally minded teachers who have been introducing new materials and new methods of learning that have not been reported. Each of us knows of several such cases. These are usually associated with some one person who has been dissatisfied with past results and has sought a different approach, either to bring new life to the classroom or to provide a greater depth and understanding to the mathematics of arithmetic. After nearly 25 years, the impact of such words as *meaning*, *understanding*, *significance*, *resourcefulness*, and *discovery*, is being felt, and with more teachers having had four and more years of training, it is to be expected that progress, or at least change, should be made.

The contribution of many of these experiments may be more in the spirit of discovery than in the significance of the mathematics learned. That must still be evaluated. But the spirit of discovery fostered by any program, whether new and experimental or with more traditional materials, constitutes the true essence of mathematics and may be more valuable

to the learner twenty years hence than his mastery of long division.

A time for decision

Let us point out some of the decisions inherent in the previous presentation. But first we are faced with the important consideration of *WHO* shall make the decisions and what criteria they shall use in deciding what mathematics shall be taught, at what levels, and to whom taught, and what modes of learning are most fruitful at the several levels and with the several kinds of students one finds in an elementary school. Now is the time for the mathematician, the teacher of young people, the teacher-educator, the parent, and the psychologist to step into the situation and work co-operatively in making decisions. This is no time for the "blind to lead the blind," however good may be their intentions. That is, let us have decisions intelligently made by competent people. The future of our individual young and of our United States is too dear to our hearts to permit policies to be formed by those who shout loudest or happen, for the moment, to be placed in positions of control—unless they perchance have the necessary background to lead.

In terms of the aims and goals to be achieved, we ask:

1 Just what should an elementary-school student gain from learning mathematics? What is the answer in terms of his life as a youngster and his later life as an intelligent participant in society?
2 What is the relative importance of studying mathematical structures and relationships versus learning to compute and to solve the common problems?
3 Is there a common set of goals for all students? What differentiation should be made in terms of ability, interest, and potential futures for different types of people?

In terms of curriculum content let us ask:

1 What criteria should be used in the selection of "newer topics" in terms of values of the topics and the grade placement thereof? For example, I recently found that a sixth-grader could learn to solve problems, such as "What is the maximum rectangular area that can be enclosed by 100 yards of fencing?" by the graphic method and the use of the simple derivative. These maxima and minima problems seem to appeal to many youngsters of this age. But shall we teach simple mechanical calculus in grade 6?
2 What conventional topics should be eliminated or greatly curtailed? For example, computations with common fractions and applications of percentage to business procedures?
3 In terms of mathematical understanding and for performance in our society, what mathematical concepts, principles and relationships are important? What relative importance should be placed upon concepts versus skills of performance?
4 What is the role of applied mathematics in the elementary school? That is, should the program stress applications or merely develop principles and procedures? In terms of method, should the one precede the other? Should they be interwoven?
5 Should the mode of learning mathematics be considered an important feature of the content? That is, are there modes of learning that are as important an acquisition as perhaps the ability to add who'e numbers?
6 Should both the language of mathematics and the concepts be viewed as gradual developments with a sophisticated level reached considerably later than at the introductory stages? Just how necessary and important is a precision of expression in the elementary school?

7 Likewise, are there stages of sophistication in logical reasoning that should be observed in the elementary school?
8 Since a program depends somewhat upon the time devoted to it, how much time should be devoted to mathematics at the several grade levels? If it is proposed to increase the amount of time, how will this increase be arranged?
9 How shall the computing machine be used in the elementary-school program?
10 What different content and levels of achievement should be established for different levels or kinds of students?

In terms of the teacher and modes of learning we inquire:

1 In addition to the personal and professional qualities of a good teacher, what background in mathematics should a teacher possess? What program is suggested for the education and the re-education of the thousands of current teachers, many of them unconcerned, who know little mathematics?
2 Is the learning process essentially the same for all normal children or is there an essential difference between the "slow learner" and the one who grasps ideas quickly?
3 What is the proper role of various aids to learning, such as manual and visual aids? TV? Films? Textbooks? How should a school use the new "learning machine"? Is there a danger that media of learning may control content and goals?
4 Are certain topics better learned by proceeding from the particular to the general, and others in reverse order?
5 Since the learning of mathematics is individual for the student, what procedures can be devised for making this efficient for a group of 30 within one class? What values are there in such programs as subgrouping, treating mathematics as a special subject similar to art and music in many schools, and in having special teachers?
6 What values can any procedure gain from the concept of "programmed learning"?

Conclusion

Yes, "mathematics is queen of the sciences and arithmetic is queen of mathematics." There are many decisions to be made. It is timely that we face these decisions. *Who* shall make the decisions? I can make them. You can make them. But let us try to obtain intelligent decisions. Any program that omits:

1 what I have called the essence of mathematics;
2 computations and procedures; and
3 application to the social, physical, cultural, and economic world,

falls short of its obligation to our young people.

I challenge the National Council to take the initiative and lead the way toward intelligent decisions or, at least, to issue a guide for making such decisions. School people are seeking advice and leadership. Now is the time for decision.

Numbers, sets, and counting*

TRUMAN BOTTS

University of Virginia, Charlottesville, Virginia

Professor Botts is a member of the mathematics department of the University of Virginia. He was also a member of both the 1960 and 1961 summer writing teams for the School Mathematics Study Group, elementary-school mathematics.

Numbers and counting have been used by mankind and have formed part of his elementary education for many centuries. *Sets*, that is, collections of things, have been with us for just as long. As a matter of fact, the first use we learn to make of numbers and counting is in determining the *size* of a set or collection of objects, that is, *how many* objects the set has as members.

The mathematical theory of sets, however, goes back less than a hundred years and has seen most of its tremendous development within the twentieth century. There is no doubt that set theory is of great importance for mathematics today. Furthermore, the explicit study of sets as such is beginning to appear, at least experimentally, in the elementary-school curriculum, and as early as the first grade. So we may expect some appreciation of what set theory in mathematics is about and what it's for to become of increasing value to elementary-school teachers.

Now to acquire a genuine appreciation of set theory (or any other part of mathematics!) it is not enough merely to chat about it and its history "from the outside." There simply isn't any substitute for diving in and examining a significant bit of set theoretic mathematics, even if it's only a tiny bit.

Let us stick to some very familiar—but not trivial—sets or classes, namely, various classes of numbers. Let us also stick to only one aspect of these sets of numbers, namely, their "size" as determined by counting. Our objective will be to arrive at a significant fact about numbers that comes out of the set theory point of view.

Infinite sets

The so-called *counting numbers* 1, 2, 3, . . . (in mathematics, ". . ." means "and so on") are familiar to every school child. So is the process of counting: to count a set of objects amounts to pairing the number 1 with one of the objects in the set, then the number 2 with another object in the set, then the number 3 with still another object in the set, and so on. If in this process 7 happens to be the last counting number paired with an object in the set, and there are no objects in the set left unpaired with numbers, then we say that the set has just 7 members.

We also say that the counting, or pairing process, has established a *one-to-one correspondence* between the set of the first 7 counting numbers and the given set: one object in this set to each of these counting numbers, and one such counting number to each object in the set.

A set is called *finite* if it has 1 member,

* Adapted from an address delivered by invitation to an elementary-mathematics sectional meeting at the Annual Meeting of the National Council of Teachers of Mathematics on April 6, 1961, in Chicago, Illinois.

or 2 members, or 3 members, or any other counting number of members. A set is also called finite if it has no members at all, that is, if it is *empty*. If neither of these things happens, the set is called *infinite*. The simplest example of an infinite set is the set of *all* counting numbers. Any set that can be placed in one-to-one correspondence with the set of all counting numbers is also infinite; such a set is called *countably infinite*. Of course the set of all counting numbers can obviously be placed in one-to-one correspondence with itself, so it is a countably infinite set.

Some countably infinite sets

Let's look at some other examples of countably infinite sets. The set whose members are the counting numbers together with zero is usually called the set of *whole numbers*. The following one-to-one correspondence shows at once that this set is countably infinite.

Whole numbers

0	1	2	3	4	5	· · ·
↕	↕	↕	↕	↕	↕	
1	2	3	4	5	6	· · ·

Counting numbers

There is something interesting to be noted here. All counting numbers are whole numbers, but one whole number, 0, is not a counting number. Thus the set of counting numbers forms just a *part* of the set of whole numbers. Yet these two sets are of the *same size*, in the sense that they can be placed in one-to-one correspondence. This paradoxical-sounding property, that *a part of a set can be of the same size as the entire set* (in the sense of one-to-one correspondence) can be shown to be characteristic of infinite sets: all infinite sets have this property, but no finite sets do.

What can we say about the set of all *integers*

$$\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$$

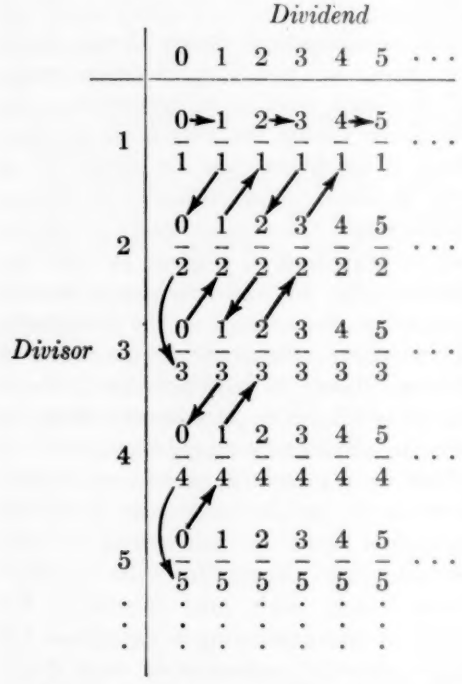
positive, negative, and zero? Is this set also of countably infinite size? Yes, because we can set up a one-to-one correspondence between this set and the set of counting numbers by starting with zero and then taking positive and negative integers alternately:

Integers

0	1	−1	2	−2	3	−3	· · ·
↕	↕	↕	↕	↕	↕	↕	
1	2	3	4	5	6	7	· · ·

Counting numbers

The quotients of whole number dividends by counting number divisors are what we call the *fractional numbers*. Do the fractional numbers form a countably infinite set? To see that they do, let us write them out in a row-and-column array in which the row tells the divisor and the column tells the dividend.



Of course, the same number is repeated, with different names, in different places in this array: for example, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, etc. We can place the set of fractional numbers in one-to-one correspondence with the set of counting numbers by following a back-

and-forth diagonal path through this array (agreeing to skip repeats) as indicated above. Following the arrows in this diagram, we find that fractional number $\frac{0}{1}=0$ corresponds to counting number 1; fractional number $\frac{1}{1}=1$ corresponds to counting number 2; fractional numbers $\frac{0}{2}$ and $\frac{0}{3}$ are skipped as repeats (since $0=\frac{0}{1}=\frac{0}{2}=\frac{0}{3}$ has already been counted); fractional number $\frac{1}{2}$ corresponds to counting number 3; fractional number $\frac{2}{1}=2$ corresponds to counting number 4; fractional number $\frac{3}{1}=3$ corresponds to counting number 5; and so on.

Fractional numbers

$$\begin{array}{ccccccc} \frac{0}{1} & \frac{1}{1} & \frac{1}{2} & \frac{2}{1} & \frac{1}{3} & \cdots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ 1 & 2 & 3 & 4 & 5 & \cdots \end{array}$$

Counting numbers

Thus the fractional numbers also form a countably infinite set.

The *rational numbers* are just the fractional numbers and their negatives. To see that they too form a countably infinite set, we must set up a one-to-one correspondence between the set of rational numbers and the set of counting numbers. We do this by starting with zero and then taking alternately in order the fractional numbers and their negatives (we used this same trick for the integers, recall).

Rational numbers

$$\begin{array}{ccccccc} \frac{0}{1} & \frac{1}{1} & -\frac{1}{1} & \frac{1}{2} & -\frac{2}{1} & \cdots \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\ 1 & 2 & 3 & 4 & 5 & \cdots \end{array}$$

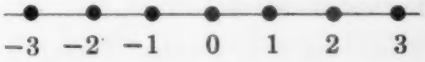
Counting numbers

In this way, we see that the rational numbers form a countably infinite set.

The real numbers

So far, all the sets of numbers we've considered—the counting numbers, the whole numbers, the integers, the fractional numbers, and the rational numbers—have turned out to be countably infinite sets. What we are going to see shortly is that there is a set of numbers, the so-called *real numbers*, so large that it is *un*countably infinite. That is, the real numbers will turn out to form a set too large to be placed in one-to-one correspondence with the set of counting numbers. This (and its corollary that irrational numbers exist) is the significant set theoretic fact about numbers that we are aiming for.

Let us begin by reviewing briefly the representation of numbers by points on a line. As you know, the successive integers may be represented by successive points equally spaced along a line, thus:



The other rational numbers are represented by appropriate points in between: $\frac{1}{2}$ by the point half-way between the points representing 0 and 1; $\frac{8}{3}=2\frac{2}{3}$ by the point two-thirds of the way from the point representing 2 to the point representing 3; and so on in familiar fashion. The points representing the rational numbers are, so to speak, spattered arbitrarily close together on the number line. You can see this for the segment of the number line between any two successive integer points by considering only those points which divide this segment into halves, then fourths, then eighths, then sixteenths, etc., since all such points represent rational numbers. Thus, arbitrarily close to any point on the number line there is a point representing a rational number. Since this is true, it might seem at least conceivable that *all* points on the number line represent rational numbers. We are going to see, though, that this is not the case.

The numbers representable by points on the number line are what are called the *real numbers*. All rational numbers, being representable by points on the number line, are real numbers. If, conversely, all real numbers were rational numbers, the set of real numbers would be the same as the set of rational numbers, a countably

infinite set. Therefore, if we can show that the real numbers do *not* form a countably infinite set, it will follow that there must be some real numbers which are not rational numbers. Such numbers are called *irrational numbers.*

Now how do we go about showing that the real numbers do not form a countably infinite set? We shall do this by a so-called indirect argument; that is, we shall show that the supposition that the real numbers *do* form a countably infinite set leads to a contradiction and hence this supposition is wrong.

Decimal numerals

In showing this, we shall find it convenient to use the decimal numeral representations of real numbers.

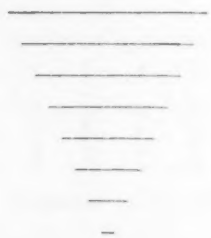
As you know, decimal numerals may be "nonterminating." A simple example is the decimal numeral representation

$$.3333333 \cdots$$

of the number $\frac{1}{3}$. Such a nonterminating decimal really corresponds to an infinite sequence of better and better approximations to the given number by numbers with terminating decimal representations. In this example, the sequence of approximations is

.3,
.33,
.333,
.3333,
.
.
.

The first approximation, .3, locates the number $\frac{1}{3}$ as being at least as big as .3 and not bigger than .4. On the number line this says that the number $\frac{1}{3}$ (or rather the point representing it) is located as lying somewhere on the segment of length .1 extending from .3 to .4. The second approximation, .33, locates $\frac{1}{3}$ more precisely as being at least as big as .33 and not bigger than .34. On the number line this says that the number $\frac{1}{3}$ is located as lying somewhere on the segment of length .01 extending from .33 to .34. The third approximation, .333, locates the number $\frac{1}{3}$ still more precisely as being at least as big as .333 and not bigger than .334. On the number line this says that the number $\frac{1}{3}$ is located as lying somewhere on the segment of length .001 extending from .333 to .334. And so on. Each of these successive segments of the number line contains all the succeeding ones, and their successive lengths are shrinking down toward zero, as suggested by the picture below.



It is thus easy to believe that all these successive segments of the number line have in common just one point; and it is this point that represents the number $\frac{1}{3}$.

In the decimal numeral .333 . . . just discussed, each successive digit happens to be 3. It is important to realize, though, that no matter what the successive digits in such a decimal numeral might be (for example, .277649134 . . .), this numeral would still determine in the same way just exactly one real number.

For uniformity, we may think of a terminating decimal as being a short way of writing a nonterminating decimal which ultimately consists of just successive 0's. For example, the terminating decimal

$$.5$$

representing the real number $\frac{1}{2}$ may be thought of as a short way of writing the nonterminating decimal

$$.5000000 \cdots .$$

On the number line, this decimal describes the point $\frac{1}{2}$ as being the point common to the segment from .5 to .6, the segment from .50 to .51, the segment from .500 to .501, etc.

There is a tricky feature of decimals to be noted here: the number $\frac{1}{2}$ also has

$$.4999999\cdots$$

as another nonterminating decimal representation. On the number line this latter decimal describes the point $\frac{1}{2}$ as being the point common to the segment from .4 to .5, the segment from .49 to .50, the segment from .499 to .500, etc.

An uncountably infinite set

Now back to our indirect argument. Hoping to arrive at a contradiction, we shall suppose that the real numbers form a countably infinite set. This means, remember, that there is a one-to-one correspondence between the set of real numbers and the set of counting numbers. We may think of this one-to-one correspondence as *listing* all the real numbers in a never-ending list. At the top of the list is the real number corresponding to 1, next the real number corresponding to 2, then the real number corresponding to 3, and so on indefinitely, *each real number occurring somewhere in the list.*

For convenience in writing their decimal numerals, let us confine attention to the real numbers between 0 and 1. From the supposition that the set of all real numbers is countably infinite it certainly follows that the set of all real numbers between 0 1 is countably infinite too, so that this latter set may be placed in one-to-one correspondence with the set of counting numbers.

Let us picture this one-to-one correspondence as written down as follows: In a column on the left we write the counting numbers, and in a parallel column on the right we write the decimal numerals for the corresponding real numbers between 0 and 1. (Where there is a choice, we may as well agree always to use the decimal numerals which ultimately consist of successive 0's rather than those which ultimately consist of successive 9's.)

The first few entires in the list might look something like this:

Counting numbers		*Decimals for real numbers between 0 and 1*
1	⟷	· 6 2 9 4 4 · · ·
2	⟷	· 3 5 4 1 6 · · ·
3	⟷	· 7 7 5 4 3 · · ·
4	⟷	· 2 8 4 7 4 · · ·
5	⟷	· 5 1 8 9 1 · · ·
·		·
·		·
·		·

The particular decimals we happen to have written down as illustrations are not important. What *is* important is that according to our supposition, *every conceivable such decimal occurs somewhere in the list* (except, by agreement, for those decimals consisting ultimately of successive 9's).

We are now going to arrive at a contradiction by constructing a new such decimal which is *not on the list after all.* The general idea for constructing this new decimal is as follows: We make the new decimal differ from the *first* decimal on the list in its *first* digit; we make it differ from the *second* decimal on the list in its *second* digit; we make it differ from the *third* decimal on the list in its *third* digit; and so on. In this way the new decimal has to differ at some point from *every decimal on the list,* and hence cannot itself be on the list, a contradiction.

There are many systematic schemes for constructing such a new decimal. One scheme is this. If the digit in question of the decimal on the list is 5, we change it to, say, 3 to get the corresponding digit of the new decimal; and if the digit in question of the decimal on the list is *not* 5, we change it to 5 to get the corresponding digit of the new decimal. In our particular illustration, the new decimal would then begin thus:

$$.5\ 3\ 3\ 5\ 5\cdots$$

(Note, incidentally, that the rule for constructing this new decimal automatically insures that it won't ultimately consist of successive 9's, since there are no 9's in it.)

Conclusion

We have now shown how it is that a contradiction necessarily results from the supposition that the real numbers form a countably infinite set. Therefore (since they certainly don't form a *finite* set!) the real numbers must form an *uncountably infinite* set. This is the significant set theoretic fact that we have set out to prove.

Recall that it now follows that not all real numbers can be rational numbers, since the rational numbers form a mere *countably* infinite set. That is, some real numbers must be what we call *irrational* numbers. There are other ways of proving that irrational numbers exist, but the way we have followed exhibits the interplay between the ancient ideas of number and counting and the modern idea of set.

As we read

Continued from page 273

change our attitudes is a question. However, a thorough study of such a rationale as Rivera proposes for dividing by a fraction will improve our self-confidence and indirectly, at least, our ease of performance and our ability to inspire some of our students. Ideally, a teacher ought to have several approaches to such a topic—all understood well enough to come into play in response to the variety of ways in which a student may sense a problem.

Koenker's short article on mental arithmetic will be sympathetically received by many who have been taken aback when students resorted to paper and pencil to convert 2/5 to a decimal fraction. Even 3/10 bothers some of them. Fortunately, some of the paper and pencil methods now being suggested in newer elementary materials are closer to the real thought processes than older forms were. For instance, using one of Koenker's examples, 29×6, which he suggests might best be worked as 6×30 and then take off 6 if you want the exact answer, we would now write $6(20+9)$ or $(6\times10)+(6\times10)+(6\times9)$ or maybe $6(30-1)$. This last, of course, is the one most people stumble upon when they practice mental arithmetic, and is a notation for the method suggested by Koenker.

Don't let the title or the sophistication of the approach prevent you from reading and rereading Bott's introduction to the way mathematicians since Georg Cantor have thought of the numbers necessary to match the points on a line. Some of this is very profound, yet the treatment by theory of sets is easy to follow. Matching of part of a set to all of a set, a characteristic of infinite sets, is the source of some and the answer to many paradoxes. Some of your students might like to speculate further on matching the even numbers to the odd and even numbers, thus:

1	2	3	4
↕	↕	↕	↕
2	4	6	8

or, upon such a question as, "Who will be dead longer, Julius Caesar or Abraham Lincoln?"

It is a long way back from this sort of thing to Sueltz's story of the little girl who liked curly brackets, but not as far as it seems at first glance. She was learning about a tool which the mathematician uses to maintain order among his ideas and maybe she was right when she preferred this to some parts of conventional arithmetic. Out of the mouths of babes. . . .

Sets and numbers in grade one, 1959–60*

PATRICK SUPPES *and*
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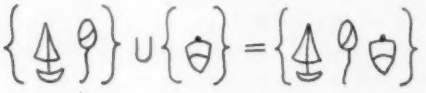
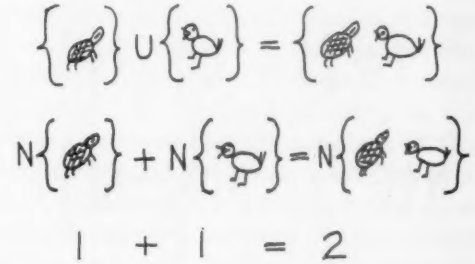
There are at least two major reasons for beginning first-grade arithmetic with the explicit introduction of the notion of set and appropriate notation for sets and operations upon them. In the first place, sets are concrete objects. Numbers are not concrete objects. Operations upon sets are more meaningful to the student than operations on numbers. The putting together of sets of physical objects, for instance, is a concrete operation. The addition of numbers is not an operation on physical objects. This is illustrated in the following example of the union of two sets.



The many exercises in the grouping of objects displayed in current workbooks are based on this idea of the union of two or more sets.

In the second place, the prior introduction of sets permits a simple and precise characterization of numbers as properties of sets. The student learns that a number is a property of a set. Continual emphasis on this definition of numbers precludes learning number symbols as meaningless symbols or addition as a meaningless operation. Numbers are properties of sets. The operation of addition of numbers is simply a general way of combining families of sets of things without paying any real attention to the things themselves. The introduction of notation for sets permits consideration of the operations of arithmetic without commitment to the particular notation of Arabic numerals. The possibility of breaking away from the reiterated use of this single notation does much to eliminate the tendency to focus on the numerals themselves without consideration of their meaning. To illustrate these points about concreteness and meaningfulness the introduction of addition at three levels is indicated below. In the first line the concrete operation of putting sets together, that is, of forming their union, is indicated. In the second line the addition of numbers is indicated; the notation of putting a large N before a set indicates the number of elements in the set. In the third line Arabic numerals are introduced.



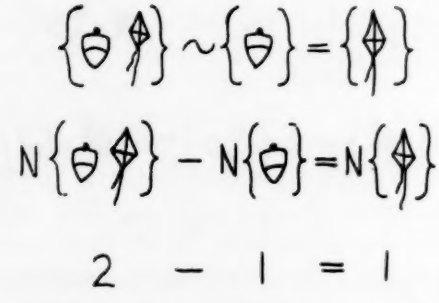
The second line of this example illustrates the fact that addition is a general way of combining sets of objects without paying attention to the objects themselves.

* The evaluative work reported here has been supported by the National Science Foundation.

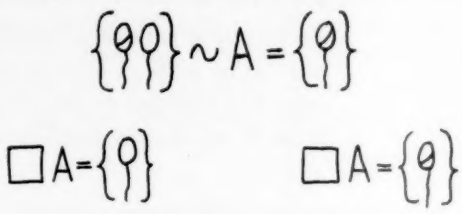
Experimental classes

From January to June, 1960, the ideas described above were tried on four first-grade classes in the Public School System of Palo Alto, California. The ideas were embodied in a workbook, *Sets and Variables, Book I*, written by the first author. The order of development of ideas is most easily indicated by a brief description of the chapters of the workbook.

Chapter I is concerned with the notion of sets itself and the identity of sets. The exercises in this chapter, as in all others, consist primarily of the student choosing one of several possible answers and indicating his choice by checking the appropriate answer box. Chapter II is concerned with the introduction of the numbers one through six as properties of sets. Drill on recognizing and using Arabic numerals is emphasized in this chapter. Chapter III deals with the union of sets, with the restriction that the combined set contain not more than six members. Chapter IV introduces addition of numbers in the manner indicated above; that is, by first considering the union of sets, then the addition of numbers with reference to sets of concrete objects, and finally by use of the Arabic numerals. Chapter V considers the difference of sets containing up to six objects, and Chapter VI contains a development of subtraction of numbers corresponding to that of addition. The three conceptual levels of the development of subtraction are indicated in the following example.



In Chapter VII the capital letters *A*, *B*, *C*, *D*, *E*, *F*, and *G* are introduced as variables to stand for sets and the lower-case letters *x*, *y*, and *z* are used to represent numbers. In the first case the students are asked to solve problems of the following type. (A check is to be placed in the box associated with the correct answer.)



In the second case variables *x*, *y*, and *z* are used in equations which are simple enough so as not to require algebraic methods of solution. For example,

$$x+3=5.$$

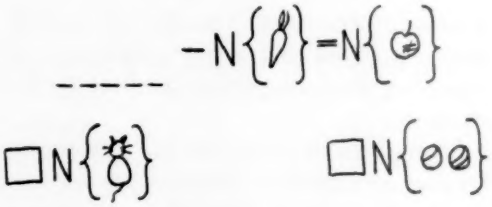
☐ $x=3$ ☐ $x=1$ ☑ $x=2$

Finally, in Chapter VIII, the empty set is introduced; in the present workbook a capital lambda (Λ) is used to indicate the empty set. The revised version will use the notation { }. Zero is introduced as a property of the empty set. Thus,

$$N\{\ \}=0.$$

(These eight chapters are comprised of 199 pages of exercises.)

Exercises of the following character are included in the workbook.



The problem is to decide what size set is needed to fill in the blank. Students who are facile in the rote operations of addition and subtraction often need considerable drill on exercises of this character.

There were 81 children in the four second-semester, first-grade classes who used the workbook experimentally. The classes were selected on a voluntary basis.

Table 1
Summary of percentage of errors—Group A

	Part I	*Part II*	*Part III*	*Part IV*	*Part V*	*Total*
No. of errors ($N=37$)	100	131	93	127	27	478
Per cent incorrect	27.0	19.7	50.3	28.6	6.9	25.8

Table 2
Summary of percentage of errors—Group B

	Part I	*Part II*	*Part III*	*Part IV*	*Part V*	*Total*
No. of errors ($N=35$)	81	65	90	121	10	367
Per cent incorrect	23.1	10.3	51.4	28.8	5.7	21.0

Monthly meetings were held with the four teachers using the workbook, at which times the teachers were asked to comment critically on the material that had been used. As a result of these criticisms the workbook was revised in a number of respects for continued experimental use during the academic year 1960–61.

Compared with the current workbooks for first-grade arithmetic the present program introduces an extensive mathematical vocabulary and notation. It is therefore of particular interest to note that, without exception, the teachers reported that their students were having no difficulty with the set-theoretical notation. They stated that the major part of the workbook would in fact be appropriate for a first-semester first-grade class, the level for which the workbook is intended. As long as the notation introduced is explicit and precise and corresponds to simple concepts, no difficulties of comprehension seem to arise.

Results

At the end of May, 1960, three of the experimental classes had completed the workbook, and the fourth class had covered all the material except for that dealing with set variables (A, B, etc.) and numerical variables (x, y, and z). The following testing procedure was devised. Two of the classes that had finished the workbook were tested on an Atronic Tutor, a teaching device; the remaining two classes were given a written test. Both groups were given the same 50 questions, covering all the material in the workbook. Henceforth the classes tested on the teaching device will be referred to as Group A, and the classes tested with a written test will be referred to as Group B.

The Atronic Tutor, which was used to test Group A, was devised by General Atronics Corporation and is at present being used experimentally. The student is shown a problem on a white card under a plexiglas window. Underneath the problem on the same card there appear from two to four possible answers. Four keys are built into the machine in front of the plexiglas window. The student makes his response by pushing the key corresponding to the answer that he believes to be the correct one. Additional cards are retained under a panel above the immediate problem, also under the plexiglas window. If the student pushes the key corresponding to the correct answer, the next card drops down. If he makes an incorrect response, nothing happens; he has to continue until he pushes the key corresponding to the correct answer. For the purposes of this experiment, 50 cards were made up, each one containing one problem with three possible answers, covering all the material in the workbook. Each of the four answer keys was covered with one of the following colors: blue, black, red, yellow. Beside each of the three possible answers on each card was placed a dot of one of the four

colors covering the answer keys. The order of correct responses was randomized. Each of the students in Group A was tested individually. The machine contains a dial which records the total number of responses made by each student, but does not record any information on which specific problems were missed or on how many incorrect responses were made on the problems which were missed. Therefore the tester observed each student while he was taking the test and recorded his performance on each of the 50 problems. The amount of time required by each student to complete the 50 problems ranged from 10 to 55 minutes.

Group B was given a written test containing exactly the same problems given to Group A. An answer box was placed beside each of the three possible answers to each problem. The two classes comprising Group B were tested separately; the members of each class took the test simultaneously. The length of time needed by each student to complete the written test ranged from 15 minutes to one hour.

For the purpose of evaluating the results, the 50-question test was divided into the following five parts: Part I, Identity, union and difference of sets, 10 questions; Part II, N operation, addition and subtraction of numbers, 18 questions; Part III, Set variables, 5 questions; Part IV, Numerical variables, 12 questions; Part V, Empty set and zero, 5 questions.

Tables 1 and 2 indicate the number of errors made in each group. For the purpose of comparison, only the first responses were counted in tabulating the results obtained from Group A. Clearly the most difficult part of the test for both groups was Part III, Set variables.

As it was suspected during the Group A testing procedure that the subjects were having difficulty adjusting to the teaching device, it was decided to compare the results obtained from the two groups. Because one of the classes of Group B had not covered the material in Parts III and IV, the results obtained on these two groups of questions were eliminated. Only Parts I, II and V were considered. A t-test was applied to total scores on these groups. The result was significant at the .05 level. The mean number of errors per student in Group A was 6.95, and for Group B was 4.4, where the total number of items was 33. The students who took the written test seem to have been significantly more successful than those who were tested on the Atronic Tutor. This fact can perhaps be explained by the previously mentioned observation that the subjects in Group A appeared to be having some difficulties in adjusting to the machine, which was new to them, whereas the students in Group B were presented with a test identical in format with the workbook.

Clearly the most difficult part of the test for the students was that which covered the use of the set variables (Part III). It is significant to note that there was most success on Part V, the empty set and zero. The concept of zero when presented in a clear and concise fashion is easily grasped by first-grade students. The greater difficulty of Part I in comparison with Part II is perhaps to be explained by the fact that the children had had considerable training in number concepts and none in set concepts before using the book.

On the basis of these results the experimental arithmetic curriculum for 1960–61 has been changed in two major respects. The use of set variables has been dropped, and the empty set and zero have been introduced much earlier.

Some new approaches to old arithmetic

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Let's examine our arithmetic programs. Do we restrict children's growth in this area because of our own limitations? Are we afraid of arithmetic? Perhaps we can't understand some of the very things which we expect youngsters to absorb. Are we, inadvertently, undermining children's understanding of mathematics? Many school districts are devoting a great deal of time and effort to an analysis of questions such as these. Here are several ideas which can help you discover some approaches which may improve your arithmetic program.

Try the path to math!

We often forget that arithmetic is a very important branch of mathematics. Its content is not finished when the pupil closes his sixth-grade textbook for the last time. A good elementary-school teacher realizes his responsibility for the child's mathematical growth long after he has left the elementary grades. Recent reports by the Mathematical Association of America and the Joint Commission on the Education of Teachers of Science and Mathematics, stress the necessity for more mathematical know-how on the part of the common-branch teacher. A great many of the techniques used in arithmetic are mechanical, but we should not lose sight of the fact that arithmetic is an essential part of that branch of science which "treats of the measuring, relations, and properties of quantities and magnitudes." It is, literally, the path to higher mathematics!

Whenever possible, the teacher should relate arithmetical computations to actual problems. This does not mean that every single algorism must be related to a social situation. One can hardly imagine a "practical" example in children's lives which would call for a computation such as $3.03 \times .05$. Nevertheless, the explanation that this multiplication represents finding a fraction (5/100) of the mixed number (3 and 3/100) would result in the type of insight which can be applied in future mathematics experiences. We should not fear mathematics. Many textbooks on arithmetic methods point out the many fascinating mathematical experiences which a simple knowledge of arithmetic can provide.

The teacher of arithmetic must have an eye on the future as well as the pupil's past. It is not enough to limit our instruction to the minimum indicated in the text. The text represents a bare outline of course content. We should equip ourselves so that we can explain arithmetic as a valuable tool, not only for learning the answers to questions with which we are faced, but as a means of approaching quantity relationships so that *new questions* can be asked.

The early years count

Everything about arithmetic revolves around the process of counting. Addition, multiplication, subtraction, and division are merely special adaptions of the basic

technique of enumeration. Early man resorted to matching to develop quantity concepts. He did not have names for his numbers and therefore could only compare one quantity with another by means of one-to-one matching. The first man to develop the idea of giving quantities consistent names which would be repeated in series must have been a great genius. Might this explain Junior's eagerness to learn to count "up to a million"? Unfortunately, parents and teachers sometimes seem to lose sight of the fact that we can do much better than primitive man and must develop sound mathematical concepts at a much faster pace than might exist in a cave-man school!

When Junior comes to kindergarten with the ability to count until he is stopped (by violence!), the teacher should capitalize on the premium that the child obviously places on mathematical prowess by encouraging him to develop power through *understanding*, rather than through mere gibble-gabble. Don't be overimpressed by the child who is glib. It is more important for the kindergartener to "know" the numbers from 1 to 5 than to count up to 100. Junior should know that 3 apples are more than 2 apples; that 4 books and 1 book are 5 books; that 3 pencils are 2 less than 5 pencils, and so on. Make every effort to develop quantity concepts in children—particularly in those who show an interest in numbers. It is wrong to give a child the impression that arithmetic ability is merely the memorization and repetition of meaningless facts. This does not mean that we should discourage the development of counting ability; rather, it means that we should couple this ability with real understanding of its uses.

Don't "take it away"!

Perhaps the easiest concept in subtraction is the one which is usually described in the "take-away" example. The teacher can illustrate very effectively that if Junior has 5 marbles and we take away 3 marbles, he has 2 marbles left. In this case, the take-away concept is effective. But there are at least two further concepts in subtraction with which the elementary-school pupil must be familiar. It is not very obvious to children that if Mary is 11 years old and Tom is 5 years old, we can determine the difference in their ages by "taking away" 5 from 11. In this type of situation, the subtraction process is used to find the difference between weights, heights, birth dates, and the like. When we subtract, we are *comparing quantities* to find out the extent of the difference between them. Another situation in which the "take-away" concept does not apply is the case where the additive method is called for. If Junior has 9 cents and wants to buy a book that costs 15 cents, he would hardly expect that he should "take away" his 9 cents from the 15 cents that he wants—in order to determine how much more money he needs so that he can make his purchase. Teachers must explain that in situations where we must determine "how much more" we need to reach a desired quantity, we are finding the *difference* between what we have and what we need. Therefore, we must subtract to find the answer.

Thorough illustration of these three types of situations must be part of the regular arithmetic program if children are to be adequately prepared for problem situations involving subtraction.

"Neither a borrower nor a lender be"

When we are teaching children to subtract 18 from 35, we are faced with the necessity for subtracting 8 from 5 in the ones' column. Many teachers explain the algorism by saying, "We need more ones in the ones' column, so we must borrow 10 ones from the tens column and pay it back before we can take away the 8." Thus, the child is left with the impression that a mysterious loan shark is employed in the process of subtraction!

Actually, it would be much easier to explain that, since we must subtract 8 from 5 in the ones' column, we must write the number 35 in a different way by *exchanging* one of the tens for 10 ones. The number 35 is now written as 2 tens and 15 ones. From this quantity we subtract 1 ten and 8 ones. This process can be illustrated effectively through the use of play money or dimes and pennies. The method is clear to children. Such terms as "borrowing" and "pay back" do not have meaning with respect to the notation used in the algorism. Consequently, the child's understanding of the subtraction process is often limited.

The tables are turned

Multiplication facts should be learned at random. Children should not have to recite the 7's table to determine the answer to 7 times 8. It is far better for them to memorize the simple fact that 7×8 is 56. This does not mean that there is no room for multiplication tables in the modern arithmetic program. On the contrary, the tables are extremely useful in teaching the relationships that exist between factors and products. Children enjoy writing the tables and analyzing them for relationships. A good teacher will point out to them that if they know the multiplication facts for all the tables from 1 to 5, they automatically know part of the tables from 5 to 10. For example, 5×7 is part of the 7's table, 4×8 is part of the 8's table, and so on.

There is a common error in the explanation of the tables, which we should avoid. If a child knows that 2×3 is 6, and he is having trouble determining the product of 2×4, we are not correct if we advise him to "add another 2 to the 2×3." Our rejection of this explanation is that 2×3 is really 3 plus 3. We should not tell the child that 2×4 is determined by adding "two" to 3 plus 3. Rather, we should explain that 2×4 is determined by the operation 4 plus 4. In other words, the 2-times table, as we usually write it, is best represented by $1+1$; $2+2$; $3+3$; $4+4$; rather than 2; $2+2$, $2+2+2$; $2+2+2+2$; etc. When we say "2×3," we mean 2 groups of 3 things. When we say "2×4," we mean 2 groups of 4 things. This is an important consideration when applied in problem solving!

Down with Gazinta!

Many of us were taught that the operation $2\overline{)8}$ is to be solved by saying "2 gazinta 8 how many times?" Actually, the question asked is not how many times does 2 go into 8 but rather how many 2's can we get from 8? Another interpretation of the question would be, "If we divided 8 into 2 equal groups, what would be the size of each group?" In the first interpretation, the children might experiment to find out how many pairs of blocks can be obtained from 8 blocks. In this case, division is really a special case of subtraction. The children "take away" 2 things as many times as possible from the 8 (answer: 4). The quotient represents the number of times that 2 can be subtracted from 8. The subtractive method of division, as this is sometimes called, tells us how many equal groups of a certain size can be separated from the original quantity.

In the partitive method of division, we are faced with the problem of separating the original quantity into a given number of groups. The quotient tells us the size of each group. Children could solve the example $2\overline{)8}$ by distributing the 8 blocks, one by one, into 2 equal groups. The quotient would represent the number of blocks in each group (answer: 4).

When children develop the "gazinta" habit, they often fail to see the relationship between the division process and the problem situation. Sometimes they will read $2\overline{)8}$ (answer: 4) as "2 *divided by* 8." This type of error can result in confusion, especially when the child begins to study fractions and is taught that 2/8 (answer: $\frac{1}{4}$) may be read as "2 divided by 8." The distinction between "2 goes into 8 how

many times?" and "How many times can we take 2 from 8?" is a subtle, but crucial point.

Don't invert; multiply

One of the most difficult operations to teach meaningfully in arithmetic involves the division of an integer by a fraction. More often than not, the teacher makes no effort to explain the rationale behind the mechanical process whereby the answer is achieved. When we are asked to find an answer for $6 \div \frac{2}{3}$, we invariably "invert and multiply." Perhaps the greatest difficulty in explaining this method arises from the fact that we actually *do not* invert and multiply at all!

Let us consider an example involving division by a decimal fraction such as $.5\overline{)2.5}$. We always "move the decimals" to remove the decimal point in the divisor. When we move the decimals, we are actually multiplying the divisor and the dividend by a quantity which *eliminates the fraction* in the divisor. In this case, we multiply both divisor and dividend by 10. We would not get a correct answer if we multiplied the divisor .5 by 10 and did not multiply the dividend as well. The example, $6 \div \frac{2}{3}$, can be written in the following form: $\frac{2}{3}\overline{)6}$. To solve such an example, we must eliminate the fraction $\frac{2}{3}$. This can be done if we multiply it by a quantity such as 3. When we multiply the fraction by 3, we must also multiply the dividend by 3. Hence, the example now becomes: $(6 \times 3) \div (\frac{2}{3} \times 3)$ or $18 \div 2$, which gives a correct answer of 9.

A slight knowledge of fractions reveals that any fraction multiplied by its reciprocal will always yield a product of 1 (e.g., $1/2 \times 2/1 = 1$; $3/4 \times 4/3 = 1$; $14/17 \times 17/14 = 1$; etc.). What we actually do when we solve an example such as $6 \div 2/3$ is to multiply the divisor and the dividend by the reciprocal of the fraction: $(6 \times 3/2) \div (2/3 \times 3/2)$. Since the fraction multiplied by its reciprocal is always 1, the division by the fraction 2/3 is no longer necessary. We do not "invert and multiply"; we merely multiply both terms by the reciprocal of the fraction.

It is important that children be given a sound basis in fraction manipulation. This is of special importance because many analogous manipulations are employed in the solution of algebraic examples.

In conclusion

We often teach "as we were taught." If you were taught poorly and you hate arithmetic, *you* will probably do a poor job teaching it. But don't surrender! We are in a fascinating age. Innumerable visual aids, manuals, and textbooks are available for those who seek them. You can improve your instructional program overnight by simply deciding that you can and *will* change your attitude about arithmetic.

Your way of saying things is strange
Your fluent phrases twist and change
How shall I find out what you mean
With true and false to choose between?

—Grace Hazard Conkling

Mental arithmetic

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The majority of arithmetic problems solved by adults and by children in their out-of-school life are solved mentally without the benefit of paper and pencil. In spite of the need for and the usefulness of mental arithmetic in daily living, most teachers devote little, if any, time to this valuable part of the arithmetic program. Some teachers still rely heavily on rigorous oral drill periods for mastering the basic arithmetical facts, but such instruction is better referred to as "mental gymnastics" than mental arithmetic.

The housewife in figuring the total cost of several purchases will mentally round off the numbers and add, thereby arriving at an approximate cost. If she is trying to find how much she will save by buying three cans of corn for 49¢ rather than paying 17¢ a can, she will either multiply 17¢ by 3 or divide 49¢ by 3 before making the purchase. The homeowner in deciding how many bags of fertilizer are needed to fertilize his lawn will mentally calculate the number of square feet in his lawn by multiplying the estimated width and length and then dividing this figure by the number of square feet which can be covered by each bag. He will use a similar mental estimation procedure in figuring out the number of gallons of paint needed to paint his home or the number of board feet of lumber needed to build a fence. The hardware store owner mentally estimates his gross profits on 30 hand lawnmowers costing him $17.95 each and selling for $24.95 by multiplying $7 by 30. A child will mentally calculate that at 3 for 10¢ he can purchase 9 candy bars for 30¢, or that each bar cost a little more than 3¢ each, or that he can save a little less than 2¢ each by buying candy bars 3 for 10¢ rather than for 5¢ each. Certainly hardly a day passes when a person does not mentally calculate an exact or approximate answer to several daily-life arithmetical problems.

Mental arithmetic, besides being socially useful, is also valuable from a mathematical point of view. In solving a problem mentally, you are forced to think through the problem without the distraction or "slowing up" process of reading the problem first and then using paper and pencil in its solution. In other words, you can focus your entire attention on the solution of the problem. If teachers devoted more time to having children solve problems mentally, the solution of problems by paper and pencil would become easier since the children have learned how to think quantitatively with numbers, a much neglected but greatly needed arithmetical ability.

On the other hand, a student may be highly proficient in finding the answers to problems by using paper and pencil, but he is something less than competent in solving the same problems mentally. The standard arithmetic algorisms are rarely, if ever, used in mental arithmetic. For

example, if the problem is to find the cost of 6 packages of notebook paper at 29¢ each, mentally the student will not rely on the algorism

$$\begin{array}{r} \$.29 \\ \times 6 \\ \hline \end{array}$$

but will round the 29 to 30 and multiply by 6, getting an approximate cost of $1.80. If the exact answer is needed, he will subtract 6¢, getting an exact cost of $1.74. To mentally find how many 4¢ stamps he could purchase for $1.36, he would not use the algorism $.04/\overline{\$1.36}$, but would solve the problem by thinking that $1.00 would purchase 25 stamps and 36¢ would purchase 9 stamps, so for $1.36 he could purchase 34 four-cent stamps.

A ten-minute daily period devoted to mental arithmetic would prove of great value in preparing a child for his present and adult out-of-school number experiences. Such a period would also help the child develop arithmetical understandings which cannot be taught by a paper-and-pencil type of arithmetic alone. Mental problems should include both those with small enough numbers for direct mental calculations as well as those which can best be solved by rounding off and approximation techniques.

A Psalm of Lamentation or The Psalm of the Empty Set

"Real Numbers" is my voodoo; I shall not pass.

It maketh me to lie down to nightmarish slumbers; it leadeth me into deep waters. It bruiseth my pride.

It seeketh to lead me out of the paths of ignorance for my pupils' sake.

Yea, though I labor and toil, my perplexities soareth to the ceiling: for it eludith me.

I have no staff of knowledge to comfort me.

It preparest a trap before me in the presence of my classmates; it anointeth my brow with sweat; my tears runneth over.

Surely sets and symbols shall follow me all the days of my life: and I shall dwell in a state of confusion forever.

—Unanimous

(Written by Mrs. Nellie Culp, *Central Grade School, Idabel, Oklahoma,* after an intensive study of the real number system as a part of a four-weeks Summer Institute Program in Mathematics for Elementary School Personnel.)

Teaching multiplication facts need not be grim

ESTHER R. UNKEL

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A more thorough and quicker mastery of multiplication facts will result if, in the initial learning period, the following two things are established:

1 discovering the facts by the children and
2 the use of concrete objects.

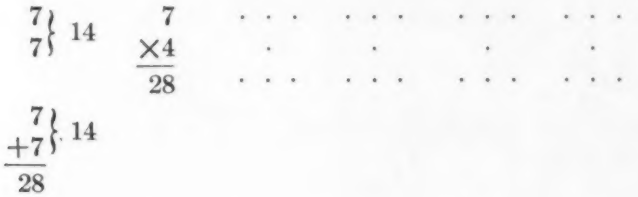
Children are placed in groups of nine to twelve, each holding 7 objects when beginning study of seven's facts. (Popsickle sticks can be purchased inexpensively.)

The first two children place 7 sticks in view.

Two 7's =	7	Two groups of 7	7
	+7	A group of 7 is	×2
	14	used 2 "times" (×)	14

The next two children clockwise will show 7 sticks each.

If two 7's = 14, then four 7's are twice this amount:



The next four children will show 7 sticks each.

If four 7's = 28, then eight 7's are double the amount:

At any time, the sticks can be pushed into one pile, and the corresponding division facts found by having a designated number of children each take 7 sticks. This will stimulate the practice of using multiplication as an aid in finding division facts. It will also help children understand the reverse process involved: in multiplication, equal groups are put together to form a whole; in division, equal groups are taken from the whole.

Encourage exploration, making no attempt to set a pattern, even though many may use the logical sequence of two 7's, four 7's, eight 7's.

Let the pupils decide how to find five groups of 7. This may be an easy fact because of their familiarity with nickels. Repeat by finding seven 7's. They may use these groupings:

Two 7's = 14	Two 7's = 14
Five 7's = 35	Two 7's = 14
Seven 7's = 49	Two 7's = 14
	One 7 = 7
Three 7's = 21	Seven 7's = 49
Four 7's = 28	
Seven 7's = 49	

They can readily see that the fewer number of 7's they know, the more detailed is the work.

It is wise to have an interruption in the instruction period at this point, followed by practice work using these facts. The remaining facts of 7 can then be found.

Three 7's = $\begin{matrix}7\\7\end{matrix}\}\ 14$ $\quad$ $\begin{matrix}14\\+7\\\hline 21\end{matrix}$ $\quad$ $\begin{matrix}7\\\times 3\\\hline 21\end{matrix}$

$\begin{matrix}7\\\hline 21\end{matrix}\}\ 7$

Six 7's = $\begin{matrix}7\\7\\7\end{matrix}\}\ 21$ $\quad$ $\begin{matrix}21\\+21\\\hline 42\end{matrix}$ $\quad$ $\begin{matrix}7\\\times 6\\\hline 42\end{matrix}$

$\begin{matrix}7\\7\\7\\+7\\\hline 42\end{matrix}\}\ 21$

Nine 7's = $\begin{matrix}7\\7\\7\end{matrix}\}\ 21$

$\begin{matrix}7\\7\\7\end{matrix}\}\ 21$

$\begin{matrix}7\\7\\7\\\hline 63\end{matrix}\}\ 21$

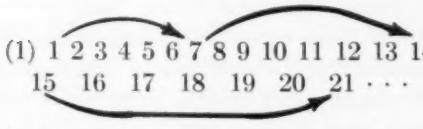
The use of concrete objects aids in seeing the "oneness" involved; nine 7's are quite a sum and many more than three 7's. We err in believing that all children are aware of this fact without manipulating objects.

Recognition that one *can find* the fact by adding several groups of 7's gives the child a feeling of security by having a method other than counting, 7, 8, 9, 10, 11, 12, 13, 14, 15, etc., until he reaches the number desired. If he does not know eight 7's, he can think, "I know four 7's (28) and another four 7's (28) and find the answer."

Is there a danger that this method will delay the mastery of the facts? It has the exact opposite effect. It gives the learner a "way out" until he has time and sufficient practice to master the fact, and it is instrumental in fostering exploration and a resultant healthy attitude not based solely on memorization.

The more capable children will enjoy the variation of working with a number line, although using a number line will have appeal and worth for all children.

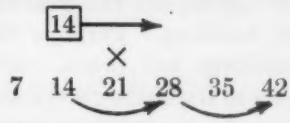
You may begin by making a simple chart:



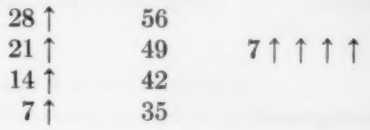
Follow this by using the 7's facts only:

7 14 21 28 35 42 49 56 63

Start at 14 and proceed by jumping one number each time:

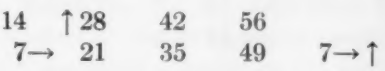


Another variation:



[Start at 7 : move up four times ↑ ↑ ↑ ↑ four 7's = 28.]

A third variation:



[Start at 7 →(21) ↑ (28) = 28.]

Fascination with the number line and the different ways of following the diagrams result in much automatic memorizing of facts after meaning has been established. It might be wise to stay with one variation of the number line until it is well known. Then the class or groups of children can devise a variation for their classmates to use, including the use of directional lines.

The use of "1" in building concepts

FRANK C. ARANITI

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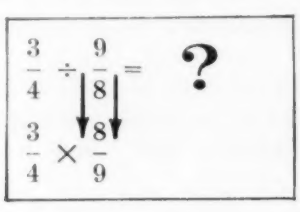
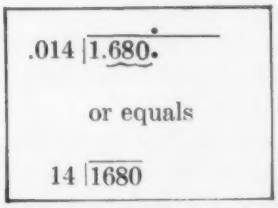
Have you ever had the same trouble as I once had in explaining why we perform the following manipulations—or do you "just teach the method"?

(A)



(B)



The careful building of arithmetic concepts must be planned and thought out as efficiently as successive stories in the con-

struction of a building if we are best to prepare students for future creative work and if we are to accomplish the goal of solid "foundation building." Perhaps the following procedures will prove useful for other teachers to try as they work with the processes of multiplication and division of fractions.

Background

Step 1. Any number divided by itself is equal to or results in 1.

$4\overline{|4}$ $\quad 10\overline{|10}$ $\quad \frac{17}{17}$ $\quad \frac{3}{3}$ $\quad 1.6\overline{|1.6}$ $\quad \frac{3.1}{3.1}$

Step 2. Any number multiplied or divided by 1 is equal to or results in the given number. For example:

$5 \times 1 = 5$ $\qquad$ $5 \div 1 = 5$

$\frac{1}{2} \times 1 = \frac{1}{2}$ $\qquad$ $\frac{5}{1} = 5$

$2\frac{1}{2} \times 1 = 2\frac{1}{2}$ $\qquad$ 5

$1\overline{|5}$

Step 3. The following relationship should also be covered.

$3 \div 4 = \frac{3}{4} = \frac{1}{4}$ of $3 = \frac{1}{4} \times 3 = 4\overline{|3}$

Step 4. Any fraction multiplied by its "reciprocal" will produce or result in 1.

Example: $\left[\frac{3}{4} \times \boxed{\frac{4}{3}} = 1 \quad \text{reciprocal of } \frac{3}{4}\right]$

Therefore in (A):

$$\boxed{\frac{3}{4} \div \frac{9}{8}} = \boxed{\frac{3}{4} \times \frac{8}{9}} = \boxed{\frac{\frac{3}{4} \times \frac{8}{9}}{1}} = \frac{3}{4} \times \frac{8}{9}$$

(Step 3) (Steps 1 and 2) (Step 2)

Note that $\dfrac{\frac{8}{9}}{\frac{8}{9}}$ is equal to 1.

In (B): $.014\overline{|1.68}$ is the same as $1.68 \div .014$ or is equal to $\dfrac{1.68}{.014}$.

Now with our problem rewritten as $\dfrac{1.68}{.014}$, the idea is to get the denominator as a whole number, which will result in a more familiar problem to the class.

To do this, we must multiply the denominator by 1000 (which is like moving the decimal point to the right 3 decimal places).

When the denominator is acted upon, we must also multiply the numerator by the same number (1000) to keep the balance and the use of our "magic" 1.

$\dfrac{1.68 \times 1000}{.014 \times 1000}$ is really $\dfrac{1.68}{.014} \times \dfrac{1000}{1000} = \dfrac{1680}{14}$

Note that $\dfrac{1000}{1000}$ is equal to 1.

Thus, the reason why $.014\overline{|1.68}$ is the same as or equal to $14\overline{|1680}$ is rather simple and digestible with the use of 1.

Research on elementary-school mathematics—1960 (concluded)

This report completes the research summary whose first portion appeared in the May, 1961, issue of THE ARITHMETIC TEACHER, pages 255–60.

In addition to this present research summary, readers will find it profitable to refer to the June 1961 issue of *The Review of Educational Research* which is devoted to the fields of "Natural Sciences and Mathematics." Sections of this issue of the *Review* relate specifically to research on elementary-school mathematics reported during the preceding three-year period.

24. Provus, Malcolm M. "Ability Grouping in Arithmetic," *The Elementary School Journal*, LX (April, 1960), 391–398.

Data for the study were derived from a sample of 19 intermediate-grade classrooms: 11 classrooms in which children at each grade level (4–6) were grouped by ability levels, and 8 control classrooms in which no ability grouping was used. In the experimental classes, the ability groups were established on the basis of performance on an adaptation of the arithmetic concepts subtest of the Iowa Tests of Basic Skills. Experimental and control classes both permitted children to move as rapidly as they could through the arithmetic curriculum, moving on to the next grade level of work at any time during the year. Findings generally favored the experimental ability-grouped classes, with a differential effect in which "the more competent pupils profited most from ability grouping."

25. Reddell, William D., and Vere DeVault, M., "In-Service Research in Arithmetic Teaching Aids," THE ARITHMETIC TEACHER, VII (May, 1960), 243–246.

This study sought to determine the relative effect of two commercial teaching aids and teacher-made aids on (1) pupils' mathematical understanding and achievement, and (2) teachers' mathematical understanding. Based on findings from 270 fifth-grade pupils and their 24 teachers, the conclusion was that for pupil groups and teachers alike, mean gains favored those using one or the other or both commercial teaching aids over those using teacher-made teaching aids.

26. Repp, Florence C. "The Vocabularies of Five Recent Third-Grade Arithmetic Textbooks," THE ARITHMETIC TEACHER, VII (March, 1960), 128–132.

Selected findings are presented from a tabulation and analysis of the vocabularies of "five widely used arithmetic textbooks of third-grade level published during the period of 1952 to June 1955."

27. Ruddell, Arden K.; Dutton, Wilbur; and Reckzeh, John. "Background Mathematics for Elementary Teachers," *Instruction in Arithmetic*, Twenty-fifth Yearbook of The National Council of Teachers of Mathematics. Washington, D.C.: The National Council, 1960. pp. 296–319.

This report gives findings from questionnaire returns from 227 mathematics educators regarding desirable content for courses in background mathematics for prospective elementary-school teachers. It

also summarizes pertinent previous research relating to this problem.

28. Schaaf, William L. "Selected Annotated Bibliography," *Instruction in Arithmetic*, Twenty-fifth Yearbook, The National Council of Teachers of Mathematics. Washington, D.C.: The National Council, 1960. pp. 320–354.

This bibliography includes some references to research studies dealing with various classified problems relating to elementary-school mathematics.

29. Schutter, Charles H. "Some Important Features of European Arithmetic Programs," THE ARITHMETIC TEACHER, VII (January, 1960), 1–8.

A condensation of findings from an analysis of arithmetic books at various grade levels from 20 European countries is offered. The author reaches five major conclusions regarding European arithmetic programs.

30. Shipp, Donald E., and Deer, George H. "The Use of Class Time in Arithmetic," THE ARITHMETIC TEACHER, VII (March, 1960), 117–121.

These researchers sought to determine the relative effect of devoting various relative amounts (75%, 60%, 40%, and 25%) of class time to developmental activities in contrast with practice work. Findings are based on data from four classes at each of three grade levels: fourth, fifth, and sixth. The study concluded that, for all ability levels: "1. There is a trend toward higher achievement, as measured by a general achievement test in arithmetic, when the per cent of class time spent on developmental activities is increased. 2. While the ideal division of class time between developmental activities and practice work could not be determined in this study, it would seem that more than 50 per cent of class time should be spent on developmental activities."

31. Stephens, Lois. "Comparison of Attitudes and Achievement Among Junior High School Mathematics Classes," THE ARITHMETIC TEACHER, VII (November, 1960), 351–356.

Answers were sought to two questions regarding 320 seventh-grade pupils and 348 eighth-grade pupils: "(1) Are the attitudes of . . . high achievers significantly different from those of . . . lower achievers? (2) Will a measure of attitudes toward arithmetic aid in selection of students for accelerated classes?" Based on use of Dutton's attitude scale, the author found: a significant (5% level) difference in mean scores on the attitude scale in favor of accelerated groups over regular groups, and in favor of accelerated groups over remedial groups; but found no statistically significant difference between regular and remedial groups. It was concluded that the attitude-scale scores ". . . would be helpful in selecting students for future accelerated classes."

32. Stephens, Lois, and Dutton, Wilbur H. "Retention of the Skill of Division of Fractions," THE ARITHMETIC TEACHER, VII (January, 1960), 28–31.

A study was designed to answer this question as a follow-up to a previous investigation by Stephens: "Is there a significant difference between the retention of the skill of division of fractions by the children taught by either the common denominator or the inversion method?" For 74 matched pairs of sixth-grade pupils who were tested with an instrument consisting of "five word problems and ten computational problems involving the different combinations of fraction, mixed number, and whole number," the study found no statistically significant difference between group means. The authors concluded that "either method is acceptable, and perhaps both should be taught in order that the process may be given meaning through the use of the common denominator method, and the students may then choose the method with which they work more easily."

33. Stright, Virginia M. "A Study of the Attitudes Toward Arithmetic of Students and Teachers in the Third, Fourth, and Sixth Grades," THE ARITHMETIC TEACHER, VII (October, 1960), 280–286.

The report gives numerous findings from administering a 25-item revision of Dutton's attitude scale to 1,023 children in grades four, five, and six, and from administering a 35-item revision of the Dutton scale to 29 teachers.

34. Suppes, Patrick, and McKnight, Blair A. *Sets and Numbers in Grade One: 1959–60.* Stanford, California: Institute for Mathematical Studies in the Social Sciences, Applied Mathematics and Statistics Laboratories, Stanford University; 1960.

Data for the study were derived from testing 72 children in four first-grade classes in which the senior author's workbook, *Sets and Variables, Book I*, was used from January to June, 1960. Children in two of the classes ($N=37$) were tested on the Atronic Tutor (a form of teaching machine); children in the other two classes ($N=35$) were tested by a conventional written test covering the same items. The research report presents a comparative analysis of frequency of error per test item for each of the two groups.

35. Tamkin, Arthur S. "A Survey of Educational Disability in Emotionally Disturbed Children," *Journal of Educational Research*, LIII (April, 1960), 313–315. [Note: A virtually verbatim report of the same study under the same title appeared in the same journal several months later: LIV (October, 1960), 67–69.]

This research was designed to test the hypothesis that "in a group of hospitalized, emotionally disturbed children, educational disabilities would predominate and that arithmetic would be lower than reading ability." Findings were based on data from administering the arithmetic and reading parts of the Wide Range Achievement Test to 22 boys and 12 girls (mean chronological ages of 8.7 and 9.4 respectively) "receiving residential treatment for emotional disorders at a psychiatric hospital." "The expectation that educational disabilities would predominate in a group of emotionally disturbed children requiring residential treatment was not borne out by the findings of this study. . . . The hypothesis that emotionally disturbed children would score higher on the Reading part than on the Arithmetic part was confirmed." The investigator contended that: "Failure to confirm the first hypothesis lent support to the theory that educational disability may be a symptom of the same condition producing the emotional problems rather than being caused by the emotional problems."

36. Townsend, Robert C. "An Experiment in Arithmetic Acceleration," THE ARITHMETIC TEACHER, VII (December, 1960), 409–411.

Here are reports on some findings on the effects of permitting a group of 44 children (in Grades 4, 5, and 6) to move ahead in arithmetic on an "accelerated" basis during the course of one school year.

37. Wallen, Norman E., and Vowles, Robert O. "The Effect of Intraclass Ability Grouping on Arithmetic Achievement in the Sixth Grade," *Journal of Educational Psychology*, LI (March, 1960), 159–163.

Based on data from 112 sixth-grade children, this research sought to compare "the achievement outcomes of intraclass ability grouping vs. nongrouping procedures in arithmetic instruction where the basis of grouping is the students' arithmetic achievement level." During the course of one school year, no significant difference in achievement for grouping and nongrouping procedures was found.

38. Weaver, J. Fred. "A Bibliography of Selected Summaries and Critical Discussions of Research on Elementary School Mathematics," THE ARITHMETIC TEACHER, VII (Nov., 1960), 364–366.

This bibliography is restricted to summaries and critical discussions published since 1940.

39. Weaver, J. Fred. "Research on Arithmetic Instruction—1959," THE ARITHMETIC TEACHER, VII (May, 1960), 253–265.

This is the 1959 counterpart of the present summary.

The following three references do not deal expressly with elementary-school mathematics, but each does include some information regarding the effect of a plan of grouping on arithmetic achievement:

40. Metfessel, Newton S. "The Saugus Experiment in Multi-Grade Grouping," *California Journal of Educational Research*, XI (September, 1960), 155–158.

41. Savard, William G. "An Evaluation of an Ability Grouping Program," *California Journal of Educational Research*, XI (March, 1960), 56–60.

42. Skapski, Mary King. "Ungraded Primary Reading Program: An Objective Evaluation." *The Elementary School Journal*, LXI (October, 1960), 41–45.

The references in this final section of the 1960 listing indicate doctoral dissertations reported in *Dissertation Abstracts*. Readers should refer directly to this source for each summary.

43. Beldin, Horace Otis. "A Study of Selected Arithmetic Verbal Problem-Solving Skills Among High and Low Achieving Sixth-Grade Children," *Dissertation Abstracts*, XXI (December, 1960), 1418.

44. Capps, Lelon Roger. "A Comparison of the Common Denominator and Inversion Method in Teaching Division of Fractions," *Dissertation Abstracts*, XXI (October, 1960), 819–820.

45. Carter, Mary Katherine. "A Comparative Study of Two Methods of Estimating Quotients When Learning to Divide by Two-Figure Divisors," *Dissertation Abstracts*, XX (February, 1960), 3317.

46. Curry, Robert Lee. "The Effect of Intelligence on the Scholastic Achievement of Sixth-Grade Children of Comparable Socio-Economic Status," *Dissertation Abstracts*, XX (April, 1960), 3995.

47. Friedman, Bernard. "An Evaluation of the Achievement in Reading and Arithmetic of Pupils in Elementary School Instrumental Music Classes," *Dissertation Abstracts*, XX (March, 1960), 3662–3663.

48. Guggenheim, Fred. "Effect of Social Climate on the Learning of Mathematics: The Effect Dominative and Integrative Classroom Climates Have on the Learning of Mathematics by Third-Grade Elementary-School Children," *Dissertation Abstracts*, XXI (November, 1960), 1125–1126.

49. Hoffman, Carl Bentley. "The Relationship of Immediate Recall, Delayed Recall, and Incidental Memory to Problem-Solving Ability," *Dissertation Abstracts*, XXI (October, 1960), 813–814.

50. Hunyard, Robert John. "An Experimental Study in Arithmetic and Its Contributions to the Research Department of the College of Education, Thailand," *Dissertation Abstracts*, XX (February, 1960), 3205–3206.

51. Lerch, Harold Hubert. "A Study Concerning the Adjustment of Arithmetic Instruction to Certain Individual Differences," *Dissertation Abstracts*, XXI (December, 1960), 1476–1477.

52. Lockwood, Stephen Hoyt. "An Experimental Investigation on the Retention of Gains in Academic Achievement Made by a Group of Gifted Children During a Program of Planned Acceleration in the Intermediate Grades," *Dissertation Abstracts*, XXI (September, 1960), 499–502.

53. Loughlin, Leo James. "A Study of Problem Solving Behaviors Among Children of Low, Average, and High Intelligence at 127 Months of Age," *Dissertation Abstracts*, XXI (September, 1960), 547–548.

54. McHugh, Walter Joseph. "Pupil Team Learning in Skills Subjects in Intermediate Grades," *Dissertation Abstracts*, XXI (December, 1960), 1460–1461.

55. McMahon, Della Lorraine. "An Experimental Comparison of Two Methods of Teaching Per Cent to Seventh-Grade Pupils," *Dissertation Abstracts*, XX (March, 1960), 3760.

56. Miller, Frances Pauline. "An Analysis of Sixth-Grade Pupils' Thinking Regarding Their Solution of Certain Verbal Arithmetic Problems," *Dissertation Abstracts*, XXI (September, 1960), 503.

57. Mullins, William Harold. "An

Classification of research reports

Area	*Research report*
Plans for grouping children	24, 37, 40, 41, 42, 54, 61, 62
Talented or gifted children	12, 36, 52, 57
Individual differences	13, 51
Factors related to achievement	16, 21, 31, 35, 46, 47, 48, 59
Problem solving	9, 18, 43, 49, 53, 56, 58
Mental arithmetic	20
Estimation, measurement	10
Intensive review	7
Division	32, 44, 45
Instructional aids	25, 63
Percentage	55
Proof	60
Distribution of class time	30
Attitudes	33
Preschool and Grade 1	3, 34
Teachers	1, 2, 15, 23, 64
Textbooks; teachers manuals	6, 11, 14, 19, 26, 27
Foreign countries	5, 22, 29, 50
Research summaries, etc.	4, 8, 17, 28, 38, 39

Arithmetic Program for the Superior Student," *Dissertation Abstracts*, XX (April, 1960), 4002.

58. Pace, Angela. "The Effect of Understanding on the Reorganization and Permanence of Learning: A Study in Verbal Problem Solving," *Dissertation Abstracts*, XX (February, 1960), 3224–3225.

59. Smail, Robert William. "Relationships Between Pupil Mean-Gain in Arithmetic and Certain Attributes of Teachers," *Dissertation Abstracts*, XX (March, 1960), 3654–3655.

60. Smith, Eugene Preston. "A Developmental Approach to Teaching the Concept of Proof in Elementary- and Secondary-School Mathematics," *Dissertation Abstracts*, XX (March, 1960), 3668–3669.

61. Smith, William Maurice. "The Effect of Intraclass Ability Grouping on Arithmetic Achievement in Grades Two Through Five," *Dissertation Abstracts*, XXI (September, 1960), 563–564.

62. Sommers, Mildred Emily. "A Comparative Study of Two Grouping Procedures in the Junior High School on Measures of Ability and Achievement in Mathematics and English," *Dissertation Abstracts*, XXI (November, 1960), 1115–1116.

63. Swick, Dana F. "The Value of MultiSensory Learning Aids in the Teaching of Arithmetical Skills and Problem Solving—An Experimental Study," *Dissertation Abstracts*, XX (March, 1960), 3669.

64. Turner, Richard Lee. "The Professional Skill of Teachers on the Criterion of Problem Solving: Teachers of Arithmetic Grades 3–6," *Dissertation Abstracts*, XX (June, 1960), 4591.

Areas of research interest

What aspects or phases of arithmetic were represented in the research reported during the 1960 calendar year? The following table attempts to answer this question in a concise way and also indicates the relative amount of research reported in each area of interest.

Some of the references might have been classified in several ways. However, each research report has been placed in just one category in the table above.

The numerals in the right-hand column are not frequencies but refer to specific research reports in the basic listing.

Concluding statement

The number of research reports relating to elementary-school mathematics for the 1960 calendar year is encouraging. It may be inferred (validly, and not erroneously, it is hoped) that this indicates increased research interest and activity. That is fine.

It is to be hoped that along with increased research interest and activity there will be more informative research reporting. To be most helpful, research reports and abstracts must tell their stories adequately. Continued improvement will accelerate translation of significant research findings into widespread practice.

In the last analysis, one of our greatest needs in relation to elementary-school mathematics is for well designed, well reported research on significant and crucial problems. The future will tell how well this need is being met.

"That mankind adopted the decimal system is a *physiological accident.* Those who see the hand of Providence will have to admit that Providence is a poor mathematician. For outside the physiological merit, the decimal base has little to commend itself."—TOBIAS DANTZIG, *Number, the Language of Science,* (New York: The Macmillan Company, 1954) p. 15.

Candidates for offices in The National Council of Teachers of Mathematics

The following slate of officers was approved by the Board of Directors at the Business Meeting held in Toronto August, 1961.

President

Frank B. Allen, Illinois
Donovan Johnson, Minnesota

Vice-President, Secondary Level

Eugene P. Smith, Michigan
Florence Elder, New York

Vice-President, Elementary Level

Edwina Deans, Washington, D.C.
Marguerite Brydegaard, California

Directors

William W. Matson, Oregon
Edward Begle, California
Burton W. Jones, Colorado
Isabelle Rucker, Virginia
Clarence H. Heinke, Ohio
Emil J. Berger, Minnesota

Books and materials

Arithmetic for the Modern Age, Aaron Bakst. Princeton, N.J.: D. Van Nostrand Company, Inc., 1960. Cloth, vii+341 pp., $4.95.

This is not a textbook, but a book "designed for busy adults who wish to review the subject for practical use in solving business and industrial problems." In addition to this comment which appears on the dust jacket, the author briefly expresses his belief that people will always need ability in arithmetic, in spite of the advent of high-speed computers.

The review of arithmetic that the reader can expect is of the computational parts of arithmetic. There is very little of how to solve problems. Real-life problem situations are used, and solutions are given, but the reader must work out for himself why certain processes were used to obtain the solution. In other words, Mr. Bakst has stayed closely within the mathematical part of arithmetic, only rarely showing the close relationship to the real world and to action that seem to be vital in problem solving.

On the whole, the book presents arithmetic as an interrelated structure. The decimal system of numeration is well presented. Although the presentation of carrying and borrowing seems almost devoid of meaningful relation to place-value ideas, these ideas are very well utilized in long division. Basic laws of associativity and commutativity are mentioned, but these laws are somewhat confused when applied to multiplication. Good use is made of the distributive law in multiplication.

With the exception, perhaps, of topics like logarithms and the slide rule, the content of this book is fairly traditional, as compared with current elementary and junior high school arithmetic. Perhaps the flavor of the book is best characterized by suggesting this title, "The Good Old Arithmetic for the Modern Age."

LYMAN C. PECK
Ohio Wesleyan University
Delaware, Ohio

How to Use Algebra in Everyday Life, Editors of Sterling. New York: Sterling Publishing Company, Inc., 1960. Cloth, 252 pp., $3.95.

This book is an easily read if not scholarly "how-to" book for general consumption by the not-too-mathematically-minded. At a time when there is a plethora of such material, this is better than some and no worse than others. It was written with the idea in mind that algebra can be used conveniently and interestingly in everyday activities. The authors observe that most readers fall into one of three categories: (1) those who have forgotten the algebra they once learned, (2) those who never really learned the subject, and (3) those who know the subject, but who have tucked this knowledge away in some seldom used corner of their minds. It is the members of the first and third groups who might find this book helpful. It would be of little help to anyone who did not have somewhat more than a mere speaking acquaintance with algebra.

The first four chapters, while constituting the most interesting section, are devoted to a rapid-fire refresher in basic arithmetic, algebra, geometry, and co-

ordinates. The authors have made very little attempt to do more than rekindle the spark of mathematical knowledge which may have lain dormant for a long period of time, and the reader is encouraged to spend no more time than he needs in review. No attempt is made to develop mathematical concepts beyond a level of understanding basic to an operational skill which can be turned into an asset in everyday activities. The major emphasis is placed on recalling rules, using tables, and learning short cuts.

The balance of the book is given over to the "how" of using algebra in everyday life as the title indicates. The authors touch on many and varied topics in their effort to include something of interest to almost anyone. The chapter "Algebra in Nature" covers such topics as "graphs and algebra in biology, heredity and its mathematics, sex determination, and multifactor inheritance." For those whose interests lie outside the field of nature, *How to Use Algebra in Everyday Life* offers the algebra of simple machines, roof trusses, fluid pressures, variable gears, gear ratios, gyroscopic effect, and power rating in motor vehicles, to mention only a few topics. Each topic is discussed very briefly but effectively although no effort toward profundity is made.

Under the supervision of Fred Reinfeld, the editors of Sterling Publishing Company have not written a textbook. They did not attempt to do so. The book admittedly was not aimed at helping people to become proficient mathematicians. It does not. It was written to show its readers how elementary algebra can be applied conveniently and effectively in many common situations. This it does and does well.

The book is cleverly written in nontechnical language which is well within the understanding of almost anyone who cares to pursue the subject in this setting. The diagrams are many and add much in the way of necessary illustration of the text. A subtle thread of humor is woven throughout and is augmented by some 35 strategically placed brain-teasers which are designed to challenge and amuse the reader. Taken as a whole, and in its several parts, the editors of Sterling have fulfilled their purpose.

WILLIAM H. DOGGETT
Abraham Lincoln Junior High School
Park Ridge, Illinois

Books received

Arithmetic: An Introduction to Mathematics, L. Clark Lay. New York: The Macmillan Company, 1961. Cloth, 323 pp., $4.50.

Arithmetic for Beginners, Brenda Lansdown. New York: Grosset and Dunlap, Inc., 1959. Cloth, 69 pp., $2.00.

Arithmetic for College Students, L. J. Adams. New York: Holt, Rinehart and Winston, Inc., 1961. Cloth, 262 pp.

Arithmetic for Today, Skill texts, Books 1 through 6 with Teacher's Guides, Thomas J. Durell, Adaline P. Hagaman, and James H. Smith. Columbus, Ohio: Charles E. Merrill Books, Inc., 1960. Paper, 128 to 152 pp., each $0.60.

Careers in Mathematics, Washington, D.C.: The National Council of Teachers of Mathematics, Inc., 1961. Paper, 28 pp., $0.25.

College Arithmetic, W. I. Layton. New York: John Wiley and Sons, Inc., 1959. Cloth, 200 pp., $2.97.

Concepts of Informal Geometry, Studies in Mathematics, Volume V, Richard D. Anderson. New Haven, Connecticut: School Mathematics Study Group, 1960. Paper, 272 pp., $1.00.

Essentials of Mathematics, Russell V. Person. New York: John Wiley and Sons, Inc., 1961. Cloth, 646 pp., $7.00.

Evaluation in Mathematics, Twenty-sixth Yearbook of The National Council of Teachers of Mathematics. Washington, D.C.: The National Council, 1961. Cloth, 216 pp., $3.00. Council members, $2.00.

Glossary of Arithmetical-Mathematical Terms, Bernard H. Gundlach. River Forest, Illinois: Laidlaw Brothers, 1961. Paper, 120 pp., $0.92.

Realm of Measure, Isaac Asimov. Boston: Houghton Mifflin Company, 1960. Cloth, 192 pp., $2.75.

Minutes of the Annual Business Meeting

Conrad Hilton Hotel, Chicago, Illinois
April 6, 1961

This year a change in procedure of the Annual Business Meeting was instituted. The Annual Business Meeting was scheduled to follow immediately after a general session at which time the President presented a paper entitled "Crystal Gazing." In this paper, President Phillip S. Jones gave a fine survey of the work of the Council during the past year and discussed the projects for the future. Past-President E. H. C. Hildebrandt presided at this general session.

Immediately following this general session, Dr. Phillip S. Jones, President, called the Annual Business Meeting to order at 1:45 P.M.

I. *A motion was made, seconded, and passed to approve the minutes of the meeting of April 22, 1960, as printed in the* Journals.

II. *Report of the 1961 Committee on Nominations and Elections.*

This report was presented by Dr. Oscar F. Schaaf, Chairman. In view of the election returns, the following were officially declared elected:

Vice-President for College: Bruce E. Meserve
Vice-President for Junior High School: Mildred Keiffer
Directors: Max Beberman
William T. Guy, Jr.
Julius H. Hlavaty

At the conclusion of the report, President Jones introduced the newly elected officers and directors.

III. *Report of the Recording Secretary.*

The Board of Directors has held two meetings since the last meeting of the Council in Buffalo in 1960. These meetings were as follows: Tempe, Arizona, December 26–27, 1960; and Chicago, Illinois, April 4–5, 1961. In the brief time allowed, it is impossible to relate all of the work and actions of the Board. Much of the work was routine business, receiving reports from committees, and discussing at length the work of the Council. This report merely intends to inform the membership of the major actions taken.

A. Due to a change in positions, Mr. William H. Glenn resigned as Vice-President. The Board elected Miss Eunice Lewis to fulfill the unexpired term of Mr. Glenn.

B. Dr. M. F. Rosskopf was elected to succeed Dr. W. Warwick Sawyer as Editor of *The Mathematics Student Journal.*

C. In view of the ever-increasing work and the duties of the President, a plan was approved whereby the President of the Council can have released time from his academic duties, up to the equivalent of one academic year, so that he can devote full time to the urgent matters of the Council at least during part of his administration. This type of arrangement has been discussed over a period of several years. Even though this arrangement was made during the presidency of Dr. Jones, the Board insisted that Dr. Jones take advantage of this new provision insofar as possible.

D. The Board approved the recommended changes in the Articles of Incorporation and in the Bylaws and provided the necessary procedure so that the changes in these two documents should be considered at this Annual Business Session of the Council.

E. The Board has approved the following future meetings of the Council:

1961 Summer: Toronto, Canada
1962 Annual (Spring): San Francisco, California
1962 Summer: Madison, Wisconsin
1963 Annual (Spring): Pittsburgh, Pennsylvania
1963 Summer: Eugene, Oregon
1964 Annual (Spring): Miami Beach, Florida

F. One of the most outstanding services rendered by the Council during 1960–61 was the conducting of the Regional Orientation Conferences in Mathematics. This project, under the directorship of Mr. Frank Allen, was supported by a grant from the National Science Foundation. Mr. Allen gave a detailed report to this session of the Board. In the near future, the membership will receive a brochure concerning this program.

G. The membership and the work of the Council continues to grow. The result is more income and greater demands on the Council. In view of these facts, the Board approved a budget of $345,000 for the year 1961–62. A detailed account of this budget will appear in the October issue of the official journals.

H. The Council is ending its first year under the new committee structure as described at the Annual Meeting in Buffalo. Although it has taken time and hard work to bring about the transition from a structure of some thirty committees to a plan of five major committees, the effort has been greatly justified. The change in committee structure does not reduce the number of people who are working on important assignments but merely concentrates the activities into five major groups. Each of the other standing committees of the National Council is now to be regarded as a subcommittee of one of these five major committees. Additional committees as needed will be appointed as subcommittees of these major committees. No attempt will be made to give a digest of the report of these committees now. This is partly due to the fact that some of the committee's actions are merely routine business and partly to the fact that major recommendations have not been completely resolved. From time to time, the membership will be informed of committee proposals as they are adopted. It is sufficient to state that the Council continues to move forward in all areas of its concern.

IV. *Report of the Executive Secretary.*

Mr. M. H. Ahrendt, Executive Secretary, gave the following report, which contains just a few high lights of the administrative and business activities of the Council during the past year.

A. Washington Office:

The full staff of the Washington Office during this school year has consisted of twenty persons. The office again outgrew its floor space and moved to larger quarters in September, 1960. During most of the previous school year the office had been split into two parts, with one office on the fourth floor of the NEA Building and the other on the sixth floor. The office is still divided into two parts, but they are now closer together, being on opposite sides of the corridor on the fourth floor.

Through the co-operation of the Budget Committee and the Board of Directors, the office was again able to purchase this year a number of valuable pieces of labor-saving equipment. This equipment includes an automatic mail opener, an automatic dating stamp for dating mail received, an automatic check endorser, two electric adding machines, one dictating machine, and an electric typewriter.

In July, 1960, a firm of management consultants made a reconnaissance study of the operation of the office. The report of the study characterized the office as a "healthy and well-managed operation." It stated that no problem areas were found in which large-scale studies were needed. The report made certain recommendations for our consideration as the office increases in size. A number of these recommendations have been or are being implemented within the office.

B. Membership:

The membership has continued to grow during the present school year. The present indications are that the total number of members and subscribers will increase by about 20 per cent. It is expected that the membership total on May 1, 1961, will be a little over 37,000. A study of the distribution of subscriptions among our members to the principal journals indicates that 85 per cent of the members take THE MATHEMATICS TEACHER, while 33 per cent take THE ARITHMETIC TEACHER, which means that 18 per cent of the members subscribe to both journals.

C. Circulation of Journals:

The print orders for the issues of each of the three journals for March, 1961, with the per cent of increase over the order for the same issue a year ago, are as follows:

THE MATHEMATICS TEACHER, 30,000 copies, 20 per cent increase

THE ARITHMETIC TEACHER, 17,000 copies, 29 per cent increase

The Mathematics Student Journal, 85,000 copies, 6 per cent increase

It will be noted from the above figures that THE ARITHMETIC TEACHER is growing more rapidly than THE MATHEMATICS TEACHER. This fact is true both among the membership subscriptions and the institutional subscriptions, but the growth is more rapid among the latter.

D. Financial Condition:

The financial condition of the Council continues to be very good, although it is relatively a little less favorable than it was a year ago. This is probably a result of the fact that our membership is not growing as rapidly now as it was growing a year ago. It seems likely that our total receipts during this fiscal year will be approximately $373,000. The receipts will probably exceed the expenditures by about $46,000, thus increasing our total resources by that amount. The ratio of total cash resources to total expenses has remained quite stable over a period of years. Total cash resources are equal to

about two-thirds of the total expenses for a year, giving us a "cushion" for emergencies.

The incomes from memberships and the sale of yearbooks have both increased during the year in close step with the increase in membership. However, the increase in the sale of pamphlets and supplementary publications has been less. We need to find a way to restore the vitality of our sales program for supplementary publications.

E. Publications:

The chief new publication for this year is, of course, the 26th Yearbook, *Evaluation in Mathematics*. The book is off the press and copies will be on sale at this meeting.

The following pamphlets have come off the press since the Annual Meeting in Buffalo a year ago: *The High School Mathematics Library*, 29,776 copies; *The Elementary and Junior High School Mathematics Library*, 19,200 copies; *The Number Story*, 7,000 copies; *Membership Directory*, 28,150 copies; *Careers in Mathematics*, 500,000 copies; *Vectors in Three Dimensional Geometry*, 5,000 copies; *Secret Codes, Remainder Arithmetic, and Matrices*, 7,000 copies; *Some Ideas About Number Theory*, 5,000 copies.

The following publications have been reprinted during the past year: *23rd Yearbook*, 10,000 copies; *21st Yearbook*, 3,000 copies; *Number Stories of Long Ago*, 7,700 copies; *How to Use Field Trips in Mathematics*, 6,000 copies; *How to Use Films and Filmstrips in Mathematics Classes*, 6,000 copies; *A Portrait of 2*, 5,000 copies; *Designing the Mathematics Classroom*, 3,000 copies; *How to Use Your Library in Mathematics*, 6,000 copies; *Education in Mathematics for the Slow Learner*, 6,000 copies.

F. Free Materials Mailed to Members:

The convention program booklets and *Membership Directory* are of course routinely sent free to members. About 13,800 copies of *The Elementary and Junior High School Mathematics Library* and 25,700 copies of *The High School Mathematics Library* were mailed free to individual members and institutional subscribers. The *TV Lesson Schedule* for the second semester of "Continental Classroom" was mailed to members with the cooperation of Learning Resources Institute, which provided the printed copies. The NCTM contributed its mailing list, the supplies, postage, and labor. About 25,000 copies of *Careers in Mathematics* were mailed to individual members a few weeks ago. In addition, members received numerous professional materials, some of a valuable nature, through the release of the membership list to other organizations.

G. Careers in Mathematics:

It is probably appropriate to make special mention of the pamphlet *Careers in Mathematics*, since it marks the completion of a project of much significance to the Council and to teachers and students of mathematics. This project represents the outgrowth of two separate projects initiated a number of years ago by the Council and the National Academy of Sciences-National Research Council. Without the dedicated and persistent help of numerous individuals from the latter organization, the pamphlet could not have been developed. This fine pamphlet contains "profiles" of eight young mathematicians, picturing the kind of work they do and the kind of persons they are. The pamphlet also contains an authoritative section on guidance and a list of addresses of persons and institutions to which the interested student may write for further information.

The Washington Office was able to assist with the project by taking complete responsibility for the details of production and distribution, and also by assisting with the editing, helping with the final writing, assisting with the design and art, and working out the distribution plan.

The funds which the National Academy of Sciences-National Research Council had been able to raise provided for the printing of a half-million copies. Approximately 77,000 of these were mailed singly to teachers of mathematics throughout the United States. Requests for quantities for distribution to students are now being received by your Washington Office. It has been necessary for us to devise a quota system for assigning copies so that the needs of students in all parts of the United States may be met. At the present time about four-fifths of the free quantities have been distributed. Through authorization of our Publications Committee, this pamphlet will be retained indefinitely, after the free distribution has been completed, as a sales item which teachers may purchase in any quantity.

H. Other Services:

The Washington Office has been able to assist in various ways with a number of other important and significant projects during the year.

In the fall of 1960, we made, at the request of a committee of the Congress, a study of the opinions of teachers of mathematics on the operation of the National Defense Education Act of 1958.

Most of the work involved in gathering information for the preparation of proposed amendments to the Articles of Incorporation and much of the work of arranging the amendments to the Bylaws

was done in the office.

The office was able to assist considerably with the administration of our series of eight Regional Orientation Conferences in Mathematics by handling the funds; supervising the printing and mailing of reports, stationery, and the like; and by answering correspondence.

The office also assisted with numerous other projects both of a special and a routine nature. The past year has undoubtedly been the busiest one which the Washington Office has experienced.

V. *Proposed Amendments to the Articles of Incorporation and Bylaws.*

SECRETARY'S NOTE: The proposed amendments to these two documents were prepared by a committee appointed by the Board and composed of M. H. Ahrendt, Houston T. Karnes, Philip Peak, and Milton W. Beckmann, Chairman. In addition, the services of legal counsel were employed. The committee worked independently and on two occasions worked with the Board. The final drafts of these two documents as approved by the Board were printed and distributed to the membership of the Council under the provisions of the existing Articles of Incorporation and Bylaws. Copies were also available and distributed to this session of the Annual Meeting.

A. Articles of Incorporation:

The Recording Secretary presented the proposed amendments to the membership, whereupon a motion was made and seconded to adopt the proposed amendments. Following a brief discussion, the question was called for, whereupon President Jones put the motion to a vote. The motion was unanimously carried.

B. Bylaws:

The Recording Secretary presented the proposed amendments to the membership, whereupon a motion was made and seconded to adopt the proposed amendments. Following a lengthy discussion, the question was called for, whereupon President Jones put the motion to a vote. The motion was unanimously carried.

SECRETARY'S NOTE: Copies of the newly adopted Articles of Incorporation and Bylaws are to be found beginning on page 469 of this issue.

VI. *Qualifications for Presidential Nominees for 1962.*

Motion: A motion was made, seconded, and passed that the Nominating Committee for 1962 be guided by the old Bylaws with regard to qualifications for a presidential nominee.

VII. *Resolutions.*

A committee appointed by President Jones and composed of J. Houston Banks, Harold P. Fawcett, and Burton W. Jones presented the following resolutions.

A. The "Continental Classroom":

The National Council of Teachers of Mathematics wishes to express its appreciation to those organizations responsible for and supporting the "Continental Classroom" in view of its unique contribution to the improvement of mathematics.

The recording secretary was hereby instructed to convey the appreciation of the Council to these organizations.

SECRETARY'S NOTE: These organizations are as follows: sponsors—The Conference Board of the Mathematical Sciences, Learning Resources Institute, National Broadcasting Company; donors—Bell Telephone System, E. I. Du Pont de Nemours & Company, General Foods Fund, IBM Corporation, Radio Corporation of America, The Ford Foundation, Union Carbide Corporation, United States Steel.

Motion: A motion was made, seconded, and passed to adopt this resolution.

B. Appreciation:

The National Council of Teachers of Mathematics wishes to express its gratitude to The Illinois Council of Teachers of Mathematics, The Chicago Elementary Teachers Mathematics Club, Men's Mathematics Club of Chicago and Metropolitan Area, and Women's Mathematics Club of Chicago and Suburbs for a tremendous job well done.

To Mr. Hobert Sistler, Chairman of the General Planning Committee, and to his twenty local committees of 253 members, the Council extends its sincere thanks for the staggering amount of work done in planning and preparing for this great convention of some ninety separate meetings.

The management of the Conrad Hilton is also to be commended for its friendly and efficient service in caring for the convention needs of some 3,500 persons.

The recording secretary is hereby instructed to convey to the appropriate persons the spirit of this resolution.

VIII. *Announcement.*

Professor J. Sutherland Frame, who has recently been appointed by the Conference Board of the Mathematical Sciences to direct a study of the design of facilities for mathematics in colleges and schools, made the following announcement in substance:

In connection with his new appointment, Professor Frame called upon the membership to convey to him any ideas and suggestions which might be helpful in this undertaking.

IX. *There being no further business, the meeting was duly adjourned at* 3:00 P.M.

Respectfully submitted,
HOUSTON T. KARNES,
Recording Secretary

Annual financial report

M. H. AHRENDT *Executive Secretary, NCTM, Washington, D.C.*

The report that follows gives a summary of the receipts and expenses of The National Council of Teachers of Mathematics for the fiscal year ending May 31, 1961. The increases in both receipts and expenditures were approximately proportional to our increase in membership during the year.

Readers may be especially interested in the following items. (1) For the first time in our history our total sale of yearbooks and pamphlets exceeded $100,000. (2) A significantly large expenditure was made for the printing and distribution of free materials to members. (3) The largest per cent of increase during the year in any item was in the sale of advertising space in the journals, which showed an increase of 50 per cent. This increase is probably a result of the fact that, through the stimulus provided by the NDEA, many new companies are entering the field of teaching aids for mathematics.

Receipts and expenditures of The National Council of Teachers of Mathematics for the fiscal year, June 1, 1960-May 31, 1961

June 1, 1960—Total Cash Resources		$171,793.76
RECEIPTS		
Memberships with subscriptions to THE MATHEMATICS TEACHER	$ 95,215.42	
Memberships with subscriptions to THE ARITHMETIC TEACHER	35,971.05	
Institutional subscriptions to THE MATHEMATICS TEACHER	39,326.50	
Institutional subscriptions to THE ARITHMETIC TEACHER	46,686.85	
Subscriptions to THE MATHEMATICS STUDENT JOURNAL	19,237.30	
Sale of advertising space in THE MATHEMATICS TEACHER	19,773.08	
Sale of advertising space in THE ARITHMETIC TEACHER	4,567.45	
Rental of membership list	5,868.24	
Interest on investments	3,387.49	
Net profit from conventions	3,927.20	
Miscellaneous	746.09	
Sale of publications		
Yearbooks	67,249.17	
Miscellaneous	33,331.28	
Total receipts	$375,287.12	
EXPENDITURES		
Washington Office	$121,494.72	
President's Office	4,989.76	
Vice-Presidents' Office expenses	781.44	
THE MATHEMATICS TEACHER	52,167.23	
THE ARITHMETIC TEACHER	31,229.17	
THE MATHEMATICS STUDENT JOURNAL	11,133.45	
Committee secretarial expenses	989.99	
Committee travel	4,476.39	
Subcommittee work	6,453.54	
Travel by Board members and Program Planning Committee	6,133.81	
Management survey of Washington office	750.00	
Preparation and printing of yearbooks	31,001.81	
Preparation and printing of supplementary publications	13,395.69	
Printing and distributing free pamphlets	7,482.36	
Printing and distributing 1960 Membership Directory	11,573.03	
Printing and distributing Amendments to Bylaws	1,941.51	
Storage and shipment of publications, miscellaneous	4,306.62	
Total expenditures	$310,300.52	
Increase in Cash Resources		$ 64,986.60
May 31, 1961—Total Cash Resources		$236,780.36

The 1961 budget

The Committee on Financial Policies, Budget, and Auditing for 1961 was composed of Burton W. Jones, Bruce E. Meserve, H. Vernon Price, Chairman, and M. H. Ahrendt, ex officio.

The Committee met at the Washington Office, January 24–25, 1961. During this period, the Committee had the fine services of the Executive Secretary, M. H. Ahrendt, and the availability of all the important records in the Central Office.

On the basis of the work done during this meeting, a proposed budget was prepared and submitted by mail to the members of the Board on February 15, 1961. At the Chicago Meeting, the Budget was presented by the Chairman and discussed thoroughly by the Board. The Budget as adopted by the Board, which includes a few modifications of the Budget as prepared by the Committee, is submitted below:

RECEIPTS		
Memberships (30,000 @ $5.00)	$150,000	
Memberships (5,000 @ $1.50)	7,500	
MSJ Subscriptions (75,000 @ $.30)	22,500	
Memberships (for second journal: 5,100 @ $3.00)	15,300	
Subscriptions, journals (7,500 @ $7.00)	52,500	
Subscriptions, journals (7,500 @ $6.75)	50,625	
Advertising in journals	20,200	
Interest on U.S. Treasury bonds	375	
Interest on savings accounts	6,000	
From reserves	20,000	
Total receipts		$345,000
EXPENDITURES		
Washington Office		
Executive Secretary		
Salary	$ 12,000	
Professional assistant		
Salary	8,000	
Secretarial, editorial, and clerical help		
Salaries	87,000	
General		
General office expenses	31,000	
Travel	4,000	
Special benefits (Social Security, hospitalization, and retirement)	10,000	
Total		$152,000
President		
Release of President's time	$ 7,000	
Office expenses	1,500	
Secretary	5,000	
Travel	10,000	
Total		$ 23,500
Vice-Presidents		
Office expenses	$ 1,000	
Total		$ 1,000
THE MATHEMATICS TEACHER		
Printing and mailing	$ 50,000	
Edit, Inc.	3,600	
Secretary	1,500	
Office expenses and equipment	800	
Travel	1,000	
Total		$ 56,900

THE ARITHMETIC TEACHER		
Printing and mailing	$ 27,500	
Edit, Inc.	3,300	
Secretary	2,000	
Office expenses and equipment	800	
Travel	1,000	
Promotion of THE ARITHMETIC TEACHER	1,000	
Total		$ 35,600
THE MATHEMATICS STUDENT JOURNAL		
Printing and mailing	$ 15,000	
Office expenses and equipment	500	
Secretary	1,000	
Travel	1,000	
Total		$ 17,500
Committee Expenses		
Executive Committee	$ 24,000	
Professional Standards Committee	2,000	
Professional Relations Committee	14,000	
Publications Committee	4,500	
Plans and Proposals Committee	2,500	
Total		$ 47,000
Miscellaneous Expenses		
Travel: Vice-Presidents, Directors, and Recording Secretary	$ 7,500	
Subsidization of conventions	4,000	
Total		$ 11,500
Total expenditures		$345,000

Articles of Incorporation

(As amended and adopted at the Annual Business Meeting, April 6, 1961)

1. The name of such corporation is The National Council of Teachers of Mathematics (Incorporated)

2. The object for which it is formed is to assist in promoting the interests of mathematics in America, especially in the elementary and secondary fields, by holding meetings for the presentation and discussion of papers, by conducting investigations for the purpose of improving the teaching of mathematics, by the publication of papers, journals, books, and reports, thus to vitalize and co-ordinate the work of many local organizations of teachers of mathematics and to bring the interests of mathematics to the attention and consideration of the educational world.

3. The management of the corporation shall be vested in a board of not less than three directors, or such greater number of directors as may be provided for in the bylaws of the corporation, as amended from time to time.

4. The corporation shall be exclusively a charitable, scientific, and educational corporation within the meaning of Section 501(c) (3) of the Internal Revenue Code of 1954, as currently in effect or as hereafter amended from time to time, provided, however, that the corporation shall not operate a post-secondary educational institution or vocational school, nor shall it be authorized to receive any child for care or placement apart from its own parent or guardian.

5. The corporation is not organized for profit or pecuniary benefit. It is a membership corporation without stock or stockholders. No part of the revenue or income of the corporation shall inure, either directly or indirectly, to the benefit of any member, officer, employee or any private individual or to the benefit of any corporation or organization, any part of the net earnings of which inure to the benefit of any private individual, provided, however, that this shall not prevent payment of reasonable compensation for services actually rendered to or for the corporation in effecting its purposes.

6. The corporation shall not, directly or indirectly:

a) Devote any substantial part of its activities to the carrying on of propa-

ganda or otherwise attempting to influence legislation, or participate or intervene, by the publication or distribution of statements or otherwise, in any political campaign on behalf of any candidate for public office;

b) Make any accumulation of income unreasonable in amount or duration or invest income in any manner to jeopardize the fulfillment of the objects of the corporation;

c) Engage in any activity affecting the corporation's right to full tax exemption as a charitable, scientific or educational organization under Federal laws or the laws of states or local governmental bodies;

d) Make any loan to any of its officers or directors;

nor shall it, directly or indirectly, devolve any pecuniary or economic benefit upon any member, sponsor, donor, creator, director, officer, or employee by reason or virtue of, or through:

e) The loan of any part of its income or property to any such person or organization without the receipt of adequate security and a reasonable rate of interest; or

f) The purchase of any property from any such person or organization for more than adequate consideration in money or money's worth or the sale to any such person or organization for less than adequate consideration in money or money's worth or any other transaction with any such person or organization which might result in the diversion of the income or property of the corporation.

7. If, at any time, the corporation shall cease to carry out the purposes as herein stated, all assets and property held by it, whether in trust or otherwise, shall, after the payment of its liabilities, be paid over to an organization, selected by the final Board of Directors of the corporation, which has similar purposes and has established its tax-exempt status under Section 501(c)(3) of the Internal Revenue Code of 1954 as now enacted or as it may hereafter be amended, and such assets and property shall be applied exclusively for such charitable, scientific, and educational programs.

8. The following persons are hereby selected as the Directors to control and manage said corporation for the first year of its corporate existence, viz.:

Charles M. Austin
333 North Oak Park Avenue
Oak Park, Illinois

H. E. Slaught
5548 Kenwood Avenue
Chicago, Illinois

Lillis Price
631 Lake Avenue
Wilmette, Illinois

9. The location is in the city of Chicago in the county of Cook in the State of Illinois, and the post office address of its business office is at No. 5548 Kenwood Avenue, in the said city of Chicago, Illinois.

Bylaws

(As amended April 6, 1961)

Article I
Name, Purpose, Affiliation, and Corporate Seal

A. Name. This organization shall be known as The National Council of Teachers of Mathematics (Incorporated).

B. Purpose. Its object shall be to assist in promoting the interests of mathematics in America, especially in the elementary and secondary fields, by holding meetings for the presentation and discussion of papers, by conducting investigations for the purpose of improving the teaching of mathematics, by the publication of papers, journals, books, and reports, thus to vitalize and co-ordinate the work of many local organizations of teachers of mathematics and to bring the interests of mathematics to the attention and consideration of the educational world.

C. Affiliation. This organization shall be a Department of the National Education Association. The Council shall encourage its members to become members of the National Education Association.

D. Corporate Seal. The corporate seal of the Council shall have inscribed thereon the name of the organization and the words "Corporate Seal—Illinois."

Article II
Membership

A. Any person who is interested in mathematics shall be eligible for membership in the Council.

B. The Board of Directors may establish various types of membership in the Council, determine the fee for each, and designate the publications included. The Mathematics Teacher and The Arithmetic Teacher shall be the official journals.

Article III
Officers, Board of Directors, Committees

Section I. Officers

A. Elective Officers. The elective officers of the Council shall be the president, the president-elect, and the four vice-presidents.
   1. Qualifications
      a) A nominee for president-elect must have been a member of the Council continuously during the preceding five years and have participated in the activities of the Council.
      b) A nominee for vice-president must have been a member of the Council continuously during the preceding three years, and have participated in the activities of the Council.
      c) There shall be a vice-president to represent each of the following fields: Elementary School, Junior High School, Senior High School, and College.
   2. Election
      a) The elective officers shall be elected by members of the Council by mail ballot.
      b) Two vice-presidents shall be elected each year.
   3. Tenure
      a) The president shall serve for two years or until his successor is installed and shall not be eligible for re-election.
      b) The president-elect shall serve for one year and then be installed as president.
      c) Each vice-president shall serve for two years or until his successor is installed and shall not be eligible for re-election.
   4. Compensation. Compensation, if any, for officers shall be fixed by the Board of Directors.
   5. Duties
      a) President. The president shall be the chief officer of the Council. He shall preside at all meetings of the Board of Directors and at the annual business meeting of the Council. He shall have the usual duties pertaining to his office and such other duties as may from time to time be assigned him by the Board of Directors. He shall appoint all committees, subject to the approval of the Board of Directors.
      b) President-elect. The president-elect shall be a nonvoting member of the Board of Directors.
      c) Vice-presidents
         (1) The vice-presidents shall be ranked first according to seniority in the office and then according to the number of votes received at the time of election.
         (2) In the event of the president's inability to serve or the inability of the president-elect to assume office as scheduled, the vice-presidents shall assume in the above order the responsibilities of the presidency.

B. Appointive Officers. The appointive officers of the Council shall be the executive secretary, the recording secretary, and the editors of the official journals.
   1. Appointment. The executive secretary, the recording secretary, and the editors of the official journals shall be appointed by the Board of Directors.
   2. Tenure. The executive secretary shall be appointed for a term of three years; he may be reappointed. The editors of the official journals shall be appointed for a term of three years; they may be reappointed but shall not serve for more than two consecutive terms. The recording secretary shall be appointed for a term of one year; he may be reappointed.
   3. Compensation. Compensation, if any, for appointive officers shall be fixed by the Board of Directors.
   4. Duties
      a) Executive Secretary. The executive secretary shall, working under the direction of the president, endeavor to carry out the policies formulated by the Council, the Board of Directors, and the Executive Committee. He shall serve as treasurer and business manager of the Council.
      b) Recording Secretary. The recording secretary shall keep a record of the proceedings of all business meetings of the Council and of the Board of Directors.
      c) Editors. Each editor shall be responsible for his journal in all phases except that of business management.

Section II. Directors

A. Number. There shall be nine directors.
B. Qualifications. A nominee for director must have been a member of the Council continuously during the preceding three years.
C. Election. The directors shall be elected by members of the Council by mail ballot according to a geographic plan determined by the Board. Three directors shall be elected each year.
D. Tenure. The directors shall serve for a term of three years or until their successors are installed. They shall be eligible for re-election but shall not serve for more than two consecutive terms.

Section III. Board of Directors

A. The voting members of the Board of Directors shall consist of the following: the president, the vice-presidents, and the directors. The executive secretary, the recording secretary, the editors of the official journals, and the president-elect shall be nonvoting ex-officio members.
B. Duties. The Board of Directors shall have general supervision of the activities of the

Council, supervise the expenditure of funds, fix salaries and bonds of officers, fill vacancies ad interim in any office, appoint editors of yearbooks, and make all appointments not specifically delegated to the president.

Section IV. Committees

A. The Executive Committee shall consist of the president as chairman, at least two voting members of the Board of Directors, and the executive secretary as a nonvoting ex-officio member.
   1. Appointment. The president shall appoint the members of the Executive Committee, subject to the approval of the Board. They shall serve for one year.
   2. Duties. This committee shall act for the Board of Directors when the Board is not in session and shall report all such actions at the next meeting of the Board. It shall also act in an advisory capacity to the president.

B. The president shall, subject to the approval of the Board of Directors, appoint whatever standing and special committees are deemed necessary to carry out the purposes of the Council.

ARTICLE IV
Meetings, Quorums, Rules of Order

A. The Council shall hold at least one convention per year, which shall be called the Annual Meeting, at such time and place as the Board may direct. There shall be a business meeting of the Council in conjunction with the Annual Meeting.

B. Special business meetings of the Council shall be called by the president under the authorization of a majority of the members of the Board of Directors.

C. The Board of Directors shall hold a meeting at the time of the Annual Meeting. Additional meetings of the Board shall be held at the call of the president or on the written request of two-thirds of the voting members of of the Board.

D. Notice of a business meeting of the Council shall be given to the members by the executive secretary at least thirty days prior to the date set for said meeting. Notice of any meeting of the Board other than regular meetings herein provided shall be given to each member of the Board at least fifteen days prior to the date set therefor.

E. At any business meeting of the Council a quorum shall consist of seventy-five members.

F. A majority of the voting members of the Board of Directors shall constitute a quorum for the transaction of business.

G. An announcement inviting members of the Council to suggest nominees for elective offices shall be published in each of the official journals approximately eleven months before the annual meeting at which the election result is to be announced. There shall be placed in nomination at least two persons for each elective office, and the Board shall instruct the executive secretary to mail ballots to all members at least thirty days before the Annual Meeting. A date for the closing of the balloting shall appear on the printed ballot.

H. *Robert's Rules of Order, Revised* shall govern the conduct of all business meetings of the Executive Committee, Board of Directors, and the Council.

ARTICLE V
Affiliated Groups

A. Any organized group of teachers of mathematics or any group of individuals preparing to become teachers of mathematics may petition the Board of Directors to become affiliated with the Council. The Board shall specify the conditions under which such affiliation may take place.

B. An affiliated group shall be entitled to send a delegate to the Delegate Assembly.

C. The Delegate Assembly shall be composed of representatives from affiliated groups. The Assembly shall hold a meeting at the time of the Annual Meeting of the Council, and shall serve as a forum which may make recommendations to the Board of Directors concerning activities and policies of the Council.

ARTICLE VI
Amendments

A. Amendments to these Bylaws may be made at any annual business meeting of the Council or special meeting called for that purpose, providing that due notice concerning such amendments shall have been printed in the official journals or mailed to each member at least thirty days before the date of such meeting.

Committees and Representatives (1961–1962)

SECRETARY OF THE BOARD
Houston T. Karnes, Baton Rouge, La.

EXECUTIVE COMMITTEE
Phillip S. Jones, Ann Arbor, Mich., Chairman, (1962)
Philip Peak, Bloomington, Ind. (1962)
Henry Van Engen, Madison, Wis. (1962)
Myrl H. Ahrendt, Washington, D.C. (ex officio)

Nominations and Elections Committee
(To begin and end terms in September of the years stated)
W. Eugene Ferguson, Newton, Mass., Chairman until Sept. 1961, member until Sept. 1962
Agnes Herbert, Baltimore, Md., Chairman, Sept. 1961–Sept. 1962, member until 1963
Mike Donahoe, Carmel, Calif. (1963)
Harold Fawcett, Columbus, Ohio (1962)
Catherine A. V. Lyons, Pittsburgh, Penn. (1964)
Irene Sauble, Detroit, Mich. (1963)
Lottchen Hunter, Wichita, Kan. (1962)
Lenore John, Chicago, Ill. (1961)
W. C. Lowry, Charlottesville, Va. (1961)
Nick L. Massey, Seattle, Wash. (1963)
Lyman C. Peck, Oxford, Ohio (1964)
Oscar Schaaf, Eugene, Ore. (1961)
Max Sobel, Fair Lawn, N.J. (1961)

Committee on Financial Policies, Budget, Auditing
Bruce Meserve, Montclair, N.J., Chairman (1962)
Julius Hlavaty, New York, N.Y. (1964)
Burton W. Jones, Boulder, Col. (1963)
Myrl H. Ahrendt, Washington, D.C. (ex officio)

Place of Meeting Committee
Lauren Woodby, Mt. Pleasant, Mich., Chairman (1962)
Arnold M. Chandler, Madison, Wis. (1964)
Mary McDermott, Concord, Calif. (1964)
Harry L. Phillips, Charleston, W. Va. (1964)
Ella S. Porter, Houston, Tex. (1962)
Myrl H. Ahrendt, Washington, D.C. (ex officio)

Committee on the Analysis of Special Programs
Philip Peak, Bloomington, Ind., Chairman (1962)
Alice M. Hach, Ann Arbor, Mich. (1964)
Kenneth B. Henderson, Urbana, Ill. (1963)
Z. L. Loflin, Lafayette, La. (1962)
Myron Rosskopf, New York, N.Y. (1963)

Research Committee
John Kinsella, New York, N.Y., Chairman
Kenneth Brown, Washington, D.C.
Howard F. Fehr, New York, N.Y.
Sheldon Myers, Princeton, N.J.
J. Fred Weaver, Boston, Mass.

Computer Project Committee
W. Eugene Ferguson, Newton, Mass., Chairman
Myrl H. Ahrendt, Washington, D.C.
Leon Ellsworth, New York, N.Y.
Kenneth Henderson, Urbana, Ill.
Burton W. Jones, Boulder, Col.
Phillip S. Jones, Ann Arbor, Mich.
Harley Tillitt, China Lake, Calif.

PROFESSIONAL STATUS AND STANDARDS COMMITTEE
John R. Mayor, Washington, D.C., Chairman (1962)
Frank B. Allen, La Grange, Ill. (1963)
John A. Brown, Newark, Del. (1964)
W. T. Guy, Jr., Austin, Tex. (1962)
Mildred Keiffer, Cincinnati, Ohio (1963)

Membership Committee
Mary C. Rogers, Westfield, N.J., Chairman
Pearl Bond, Beaumont, Tex.
Janet Height, Wakefield, Mass.
Lucille Houston, Racine, Wis.
Harold J. Hunt, Seattle, Wash.
Florence E. Ingham, Bartlesville, Okla.
Faith Novinger, Washington, D.C.

PROFESSIONAL RELATIONS COMMITTEE
Irvin Brune, Cedar Falls, Iowa, Chairman (1962)
J. Houston Banks, Nashville, Tenn. (1962)
Veryl Schult, Washington, D.C. (1963)
Joseph F. Senta, Minneapolis Minn. (1963)
Howard Fehr, New York, N.Y. (1964)

Affiliated Groups Committee
Eugene Smith, Detroit, Mich., Chairman (1963)
Vincent Brunner, Milwaukee, Wis. (1964)
Mary Louise Fisher, Joliet, Ill. (1964)
Mary Rogers, Westfield, N.J. (1962)
Kenneth Skeen, Concord, Calif. (1964)
Keene C. Van Orden, San Angelo, Tex. (1963)
Annie John Williams, Durham, N.C. (1962)

International Mathematics Education Committee
E. H. C. Hildebrandt, Evanston, Ill., Chairman (1963)
Howard Fehr, New York, N.Y. (1963)
Burton W. Jones, Boulder, Col. (1963)

Alfred L. Putnam, Chicago, Ill. (1962)
Ernest R. Ranucci, Newark, N.J. (1962)
Veryl Schult, Washington, D.C. (1962)

Committee on Relations with Industry
Marie S. Wilcox, Indianapolis, Ind., Chairman (1962)
Clifford Bell, Los Angeles, Calif.
George Rietz, New York, N.Y.
Edward Spacapan, Arlington Heights, Ill.

NCTM Representatives

AAAS Co-operative Committee
H. Vernon Price, Iowa City, Iowa (1964)

Conference Board of the Mathematical Sciences
Burton W. Jones, Boulder, Col. (1962)
John R. Mayor, Washington, D.C.

U.S. Commission on Mathematical Instruction
E. H. C. Hildebrandt, Evanston, Ill. (1963)
Burton W. Jones, Boulder, Col. (1964)

AAAS Council
H. Glenn Ayre, Macomb, Ill. (1963)
James H. Zant, Stillwater, Okla. (1962)

Council on Co-operation in Teacher Education
Veryl Schult, Washington, D.C. (1962)

Delegates to TEPS Meeting
Lee E. Boyer, Harrisburg, Pa. (1961)
Earle F. Myers, New Brighton, Pa. (1961)

NEA Representative Assembly
Julius H. Hlavaty, New York, N.Y. (1961)

Publications Committee
Donovan A. Johnson, Minneapolis, Minn., Chairman (1963)
Paul Johnson, Los Angeles, Calif. (1963)
Ben A. Sueltz, Cortland, N.Y. (1962)
Henry Van Engen, Madison, Wis. (1962)
Lenore John, Chicago, Ill. (1964)

Ex-officio nonvoting members
Myrl H. Ahrendt, Washington, D.C.
E. Glenadine Gibb, Editor, The Arithmetic Teacher
Robert E. Pingry, Editor, The Mathematics Teacher
Myron F. Rosskopf, Editor, The Mathematics Student Journal

Supplementary Publications Committee
Kenneth Henderson, Urbana, Ill., Chairman (1962)
Marguerite Brydegaard, San Diego, Calif. (1962)
M. Albert Linton, Philadelphia, Pa. (1964)
Eugene Nichols, Tallahassee, Fla. (1963)
Helen Schneider, Oak Park, Ill. (1963)
Daniel W. Snader, Washington, D.C. (1964)

Yearbook Planning Committee
Ben A. Sueltz, Cortland, N.Y., Chairman (1963)
J. Houston Banks, Nashville, Tenn. (1962)
Edwin C. Douglas, Watertown, Conn. (1964)
Joseph J. Stipanowich, Macomb, Ill. (1964)

Editorial Board, The Mathematics Teacher
Robert E. Pingry, Urbana, Ill., Editor (1962)
Jackson B. Adkins, Exeter, N.H.
Mildred Keiffer, Cincinnati, Ohio
Daniel B. Lloyd, Washington, D.C.
Z. L. Loflin, Lafayette, La.
Ernest Ranucci, Union, N.J.

Editorial Board, The Arithmetic Teacher
E. Glenadine Gibb, Cedar Falls, Iowa, Editor (1963)
E. W. Hamilton, Cedar Falls, Iowa
Edwina Deans, Washington, D.C.
Clarence Ethel Hardgrove, DeKalb, Ill.
J. Fred Weaver, Boston, Mass.
Marguerite Brydegaard, San Diego, Calif.
John R. Clark, New Hope, Penn.
Vincent J. Glennon, Syracuse, N.Y.

Editorial Board, The Mathematics Student Journal
Myron F. Rosskopf, New York, N.Y., Editor (1964)
Stephen Willoughby, Madison, Wis.
Mannis Charosh, New York, N.Y.

Editorial Committee, *Twenty-seventh Yearbook* (Mathematics for the Talented)
Julius H. Hlavaty, New York, N.Y., Chairman
Albert Blank, New York, N.Y.
Vincent J. Glennon, Syracuse, N.Y.
Joseph N. Payne, Ann Arbor, Mich.
Richard Pieters, Andover, Mass.
Harry Ruderman, New York, N.Y.
Henry Syer, Kent, Conn.

Plans and Proposals Committee
Robert E. K. Rourke, Kent, Conn., Chairman (1962)
Roy Dubisch, Fresno, Calif. (1962)
William Matson, Portland, Ore. (1963)
Henry Van Engen, Madison, Wis. (1964)

"The school year is being extended beyond one hundred eighty days in many districts.—*From "Schools in Our Democracy," Office of Education, U.S. Department of Health, Education, and Welfare.*

Report of the Membership Committee

MARY C. ROGERS *Chairman, Membership Committee,*
Edison Junior High School, Westfield, New Jersey

You who were present at the Thirty-eighth Annual Meeting of The National Council of Teachers of Mathematics in April, 1960, will remember the remarkable membership growth which we were able to report and the enthusiasm with which it was received throughout the convention. The combined efforts of the personnel of a great organization working toward a common goal had brought about results far beyond expectations.

Portions of this report were later published in editions of THE MATHEMATICS TEACHER and THE ARITHMETIC TEACHER and the expanded goal of 40,000 members to be reached by April 1961 announced.

What progress have we made toward the realization of this goal? May we present a few outstanding facts and leave it to you to draw your own conclusions.

The May 1, 1960, total membership was 31,194, or 194 more than the estimate for that date.

The April 6, 1961, reported total—based on the February 1, 1961, count, was 34,491—5509 less than the 40,000 hoped for that date.

By May 1, 1961, nearly *3000 additional memberships* had been reported, bringing the new total to 37,224, slightly more than had been originally anticipated for this date, but nearly 3000 less than the newly accepted goal of 40,000. This total, however, is slightly more than 93% of total goal.

Other interesting data show:

14 states and territories having reached their shares in the 40,000 goal or gone beyond.

10 states and territories with membership achievements of 90–99%.

7 states and territories with achievements of 85–89%.

1 of the seven regions has already reached 120% of new goals.

5 of the remaining 6 regions have reached 90–93% achievements.

The *total membership growth for 1960–61 has been 6030,* as compared with the 7350 increase for the previous year. This is a little disappointing, but we still think the record is quite commendable when the circumstances are known.

Since we have grown to be such a large organization, the Board of Directors has found it most difficult to transact all necessary business through direct reports from the many committees heretofore reporting to them directly. This has resulted in a complete reorganization of committees and committee services. You will recall reading about this in THE MATHEMATICS TEACHER, October, 1960 issue.

There are now five major committees supplemented by such other committees as the Board deems essential. Some of the former committees have been discontinued; others will continue their services as subcommittees of the five major committees.

At the Christmas Meeting in Tempe it was ruled that: "The Membership Committee should be continued as a subcommittee of the Committee on Professional Standards and should proceed as customary in the past until such time as further growth of the organization or change in organizational procedures make a revision of plans and procedures necessary."

As soon as reactivation of the Membership Committee services had thus become official, your Chairman contacted Dr.

John R. Mayor, Chairman of the Committee on Professional Standards, for instructions. Dr. Mayor immediately wrote expressing in reply his pleasure that the Membership Committee would be continued and offering whatever assistance we desired as a subcommittee under his direction.

The Membership Committee has been reduced in size from ten persons to seven persons. This has made necessary a regrouping of the States as to Regions and Membership Committee leadership in these Regions.

As a part of Membership reactivation, your former Membership Committee was reappointed by Dr. Jones and by the Board of Directors and was given their expanded assignments by the Chairman. Furnished with the latest membership count and analyses, they have been instructed to resume the procedures found so effective in the past—with added emphasis on:

1 An *expanded program of publicity* and an increased program of *judicious advertising* which will point out the purposes of the Council together with the *many benefits* to be gained from membership.
2 *Greater attention given to renewals* as well as to solicitation of new memberships.
3 *Closer co-operation and articulation of effort* between the Membership Committee, the Affiliated Groups, and the State Representatives.

At its meeting in Chicago, the Membership Committee set up a definite schedule of specific procedures whereby it hopes to perform a more effective service to you and to the Council. We welcome any suggestions you may have for us in bringing about this improvement. You will be hearing from us rather regularly—in line with our established schedule.

In the tables that follow you will find rather complete analyses of current membership status by States and by Regions. We believe that you will find these interesting and helpful. You will see that progress this year—although still very good—has suffered a slight setback when compared to last year's remarkable growth. We are convinced you are determined to correct this condition and make next year another banner year. Your enthusiasm and eager interest at the Chicago convention are indicative of this strong intention.

If we make another 7000 membership growth, or even 6000 growth, we will be well beyond our 40,000 goal. A reactivated Committee aided by your fine co-operation and great helpfulness will easily bring about this high objective.

Your Membership Committee thanks you most sincerely for your assistance at all times, and extends to you its best wishes for your greatest success in all of your professional endeavors.

PEARL BOND, *Beaumont, Tex.*
JANET HEIGHT, *Wakefield, Mass.*
LUCILLE HOUSTON, *Racine, Wis.*
HAROLD J. HUNT, *Seattle, Wash.*
FLORENCE INGHAM, *Bartlesville, Okla.*
FAITH NOVINGER, *Washington, D.C.*
MARY REED, *Benton Harbor, Mich.*

Ex officio members:
EUGENE P. SMITH, *Detroit, Mich.*
MYRL H. AHRENDT, *Washington, D.C.*

Chairman:
MARY C. ROGERS, *Westfield, N.J.*

"School districts are increasingly offering summer schools, with emphasis on acceleration as well as remedial work."—*From "Schools in Our Democracy," Office of Education, U.S. Department of Health, Education, and Welfare.*

Analysis of membership growth, members and subscribers, May 1960 to May 1961

	May 1960	May 1961		Goals and per cent of goals reached	
		Individuals	Totals	40,000 Goal	Per cent
Alabama	316	246	373	451	83
Alaska	19	13	24	32	75
Arizona	339	367	462	160	289
Arkansas	227	206	283	456	62
California	2,455	1,801	2,972	2,344	123
Colorado	402	400	508	480	106
Connecticut	503	390	561	608	92
Delaware	111	92	119	187	64
District of Columbia	239	232	275	491	56
Florida	740	638	847	936	90
Georgia	527	296	638	496	129
Hawaii	137	121	171	121	141
Idaho	39	27	41	48	85
Illinois	2,157	2,478	3,178	3,168	100
Indiana	916	821	1,034	1,443	72
Iowa	522	440	643	787	82
Kansas	513	495	598	827	72
Kentucky	240	210	261	328	80
Louisiana	518	333	520	736	71
Maine	137	118	145	184	79
Maryland	529	483	690	760	91
Massachusetts	867	735	1,004	1,171	86
Michigan	1,852	1,255	2,067	1,600	129
Minnesota	889	711	938	1,068	88
Mississippi	209	198	240	328	73
Missouri	549	461	611	880	69
Montana	148	117	152	160	95
Nebraska	268	220	289	440	66
Nevada	44	44	57	35	163
New Hampshire	128	109	146	171	85
New Jersey	1,152	1,000	1,387	1,496	93
New Mexico	166	113	182	235	77
New York	2,893	2,325	3,221	3,147	102
North Carolina	459	282	566	680	83
North Dakota	127	126	159	115	138
Ohio	1,345	1,340	1,689	1,971	86
Oklahoma	472	418	614	632	97
Oregon	437	330	509	355	143
Pennsylvania	1,896	1,731	2,297	2,451	94
Rhode Island	144	125	169	187	90
South Carolina	207	172	256	363	71
South Dakota	90	60	95	96	99
Tennessee	339	277	372	656	57
Texas	1,425	1,093	1,543	1,792	86
Utah	144	121	174	128	136
Vermont	71	64	80	107	75
Virginia	519	469	687	952	72
Washington	511	320	534	608	88
West Virginia	183	149	180	419	43
Wisconsin	734	645	932	1,120	83
Wyoming	83	76	107	96	111
TOTALS	29,937	25,293	35,600	38,502	92
U.S. Possessions	36	46	61	64	95
Canada	670	554	906	536	169
Foreign	551	321	657	904	73
GRAND TOTALS	31,194	26,214	37,224	40,006	93

States relatively near the 40,000 goal

	Membership increase May 1960–May 1961	*Nearness to 40,000 goal*
Alabama	57	− 78
Alaska	5	− 8
Connecticut	58	− 47
Florida	107	− 89
Idaho	2	− 7
Iowa	121	−144
Maryland	161	− 70
Massachusetts	137	−167
Montana	4	− 8
New Hampshire	18	− 25
New Jersey	235	−109
North Carolina	107	−114
Ohio	344	−282
Oklahoma	142	− 18
Pennsylvania	401	−154
Rhode Island	25	− 18
South Dakota	5	− 1
Vermont	9	− 27
Wisconsin	198	−188
U.S. Possessions	25	− 3

States and territories having reached their goals or gone beyond

(Based on 40,000 goal)

Arizona	289%	North Dakota	138%	Wyoming	111%
Canada	169%	Utah	136%	Colorado	106%
Nevada	163%	Georgia	129%	New York	102%
Oregon	143%	Michigan	129%	Illinois	100%
Hawaii	141%	California	123%		

Membership achievement 90%-99%

(Based on 40,000 goals)

South Dakota	99%	Pennsylvania	94%	Maryland	91%
Oklahoma	97%	New Jersey	93%	Florida	90%
Montana	95%	Connecticut	92%	Rhode Island	90%
U.S. Possessions	95%				

Membership achievement 85%-89%

(Based on 40,000 goals)

Minnesota	88%	Ohio	86%	Idaho	85%
Washington	88%	Texas	86%	New Hampshire	85%
Massachusetts	86%				

Leaders in membership totals

(Including subscriptions)

New York	3,221	Ohio	1,689	Minnesota	938
Illinois	3,178	Texas	1,543	Wisconsin	932
California	2,972	New Jersey	1,387	Canada	906
Pennsylvania	2,297	Indiana	1,034	Florida	847
Michigan	2,067	Massachusetts	1,004	Maryland	690

Leaders in membership totals

(Not including subscriptions)

Illinois	2,478	Michigan	1,255	Minnesota	711
New York	2,325	Texas	1,093	Wisconsin	645
California	1,801	New Jersey	1,000	Florida	638
Pennsylvania	1,731	Indiana	821	Canada	554
Ohio	1,340	Massachusetts	735	Kansas	495

Leaders in membership growth

(Including subscriptions)

Illinois	1,021	Canada	236	Maryland	161
California	517	New Jersey	235	Oklahoma	142
Pennsylvania	401	Michigan	215	Massachusetts	137
Ohio	344	Wisconsin	198	Arizona	123
New York	328	Virginia	168	Iowa	121

Leaders in per cent of growth

(Compared with May, 1960, totals)

U.S. Possessions	Nevada	Colorado
Illinois	Oklahoma	Ohio
Arizona	Wyoming	Arkansas
Canada	Wisconsin	Hawaii
Virginia	Alaska	North Dakota
Maryland		

States and territories with continuous growth

(Since May, 1960)

Alabama	Kentucky	Oklahoma
Arizona	Massachusetts	U.S. Possessions
Delaware	Michigan	Vermont
Georgia	Ohio	

Membership analysis by territory

	May, 1960		*May 1, 1961*	
	Membership and per cent toward goal of 25,000		*Membership and per cent toward goal of 40,000*	
Pearl Bond Alabama, Arizona, Louisiana, Mississippi, Nevada, New Mexico, Texas	3,017	129%	3,377	90%
Janet Height New England States, Florida, Georgia	3,117	129%	3,590	93%
Lucille Houston Illinois, Iowa, Minnesota, Missouri, North Dakota, South Dakota, Wisconsin	5,068	112%	6,556	91%
Harold J. Hunt Alaska, California, Hawaii, Idaho, Montana, Oregon, Washington	3,746	163%	4,403	120%
Florence Ingham Arkansas, Colorado, Kansas, Nebraska, Oklahoma, Utah, Wyoming	2,109	110%	2,573	84%
Faith Novinger Delaware, District of Columbia, Maryland, New Jersey, New York, North Carolina, Pennsylvania, South Carolina	7,486	125%	8,811	92%
Mary Reed Indiana, Kentucky, Michigan, Ohio, Ontario, Tennessee, Virginia, West Virginia	6,064	123%	7,196	91%

Professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE ARITHMETIC TEACHER. Announcements for this column should be sent at least two months prior to the month in which the issue appears to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D.C.

NCTM convention dates

Fortieth Annual Meeting
April 16–18, 1962
Jack Tar Hotel, San Francisco, California
Kenneth C. Skeen, 3355 Cowell Road, Concord, California

Joint Meeting with NEA
July 4, 1962
Denver, Colorado
M. H. Ahrendt, 1201 Sixteenth Street, N.W., Washington 6, D.C.

Twenty-second Summer Meeting
August 23–25, 1962
University of Wisconsin, Madison, Wisconsin
H. Van Engen, School of Education, University of Wisconsin, Madison 6, Wisconsin

Other professional dates

Arkansas Council of Teachers of Mathematics
October 6–7, 1961
Arkansas State Teachers College, Conway, Arkansas
Mrs. R. H. Jones, 1229 Dyer, Malvern, Arkansas

Women's Mathematics Club of Chicago and Vicinity
October 7, 1961
Wedgwood Room, Marshall Field & Company, Chicago, Illinois
Dr. Ruth Ballard, University of Illinois, Navy Pier, Chicago 11, Illinois

Illinois Council of Teachers of Mathematics
October 13–14, 1961
University of Illinois, Champaign-Urbana, Illinois
T. E. Rine, Illinois State Normal University, Normal, Illinois

Maryland State Teachers' Association, Mathematics Section
October 20, 1961
Edmondson High School, Edmondson Village, Maryland
W. Edwin Freeny, Woodlawn High School, Baltimore 7, Maryland

Washington State Mathematics Council
October 27–28, 1961
University of Puget Sound
Professor Ed Goman, 1500 North Warner, Tacoma, Washington

Women's Mathematics Club of Chicago and Vicinity
November 4, 1961
English Room, Marshall Field & Company, Chicago, Illinois
Dr. Ruth Ballard, University of Illinois, Navy Pier, Chicago 11, Illinois

Georgia Mathematics Council
November 10–11, 1961
Rock Eagle State Park, Eatonton, Georgia
Martha Rogers, 2908 Macon Road, Columbus, Georgia

Women's Mathematics Club of Chicago and Vicinity
December 2, 1961
Henrici's Restaurant, 71 W. Randolph, Chicago, Illinois
Dr. Ruth Ballard, University of Illinois, Navy Pier, Chicago 11, Illinois

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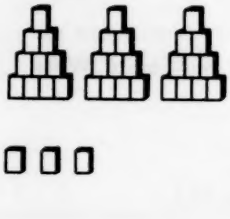

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5500 is 5 × 1100.
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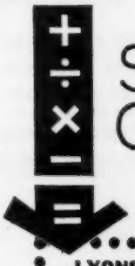

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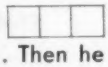
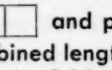
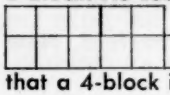
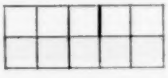
IN STRUCTURAL ARITHMETIC the pupil takes a 3-block  and a 2-block and puts them together  . Then he compares the combined length of the two blocks and discovers *for himself* that the length of the 3-block plus the 2-block can be equal only to the 5-block. He sees that . . .

 a 6-block is too large, and he sees that a 4-block is too small.

He is certain that 3 plus 2 makes 5  . The 3 and 2 have not lost their identity, and every time he looks at the blocks he *sees* that 3 and 2 are the same as 5.

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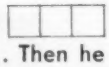
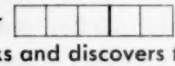
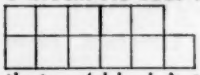
IN STRUCTURAL ARITHMETIC the pupil takes a 3-block  and a 2-block and puts them together  . Then he compares the combined length of the two blocks and discovers *for himself* that the length of the 3-block plus the 2-block can be equal only to the 5-block. He sees that . . .

 a 6-block is too large, and he sees that a 4-block is too small.

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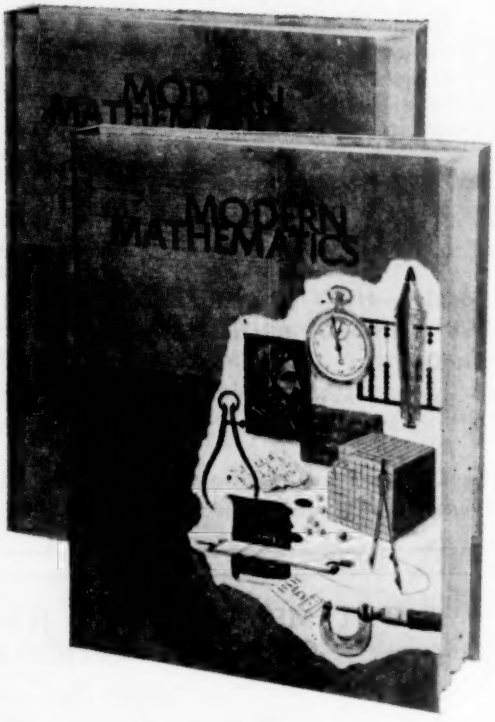

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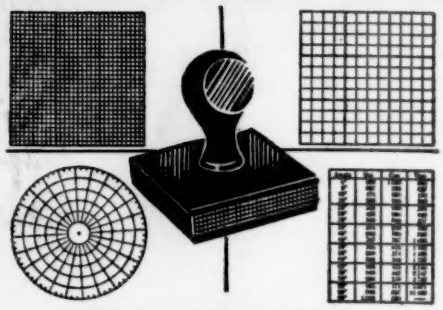

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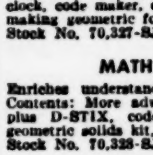

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